



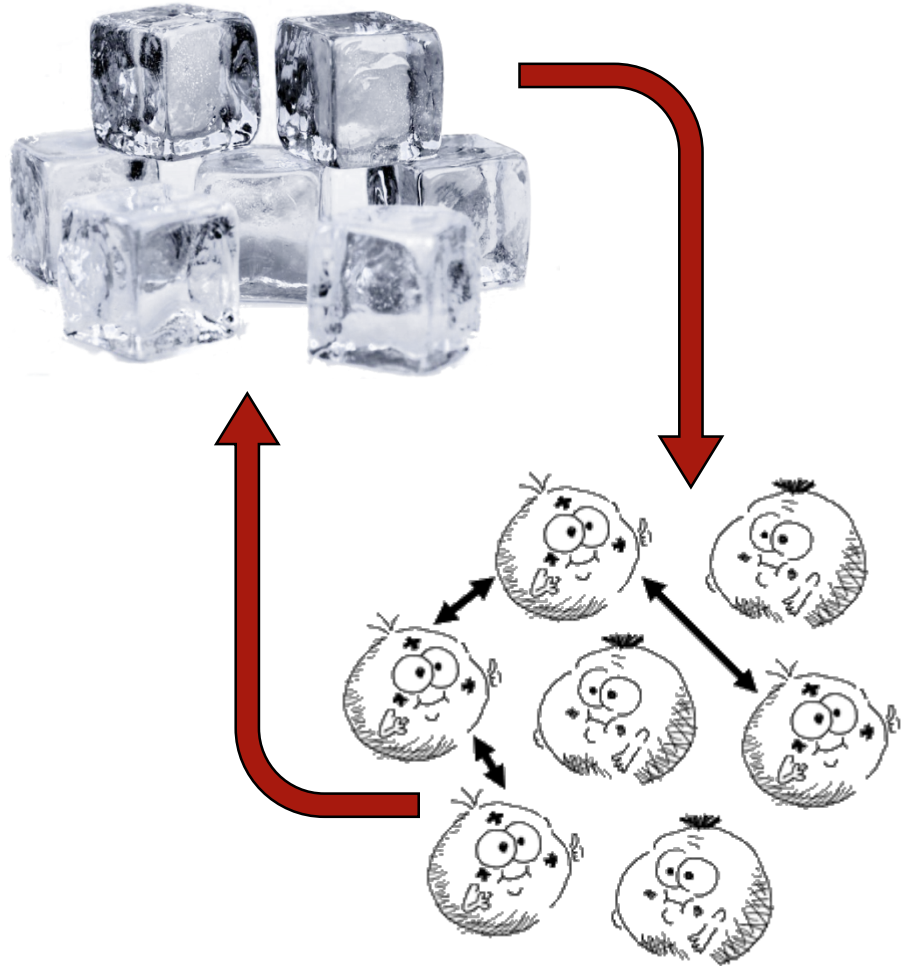
Towards understanding of the social hysteresis: an agent-based approach

(NCN 2019/35/B/HS6/02530)

Katarzyna Sznajd-Weron
Statistical **P**hysics **i**n **C**omplex **S**ystems (SPiCy) group
Department of Theoretical Physics

Why statistical physics?

Macro-scale: thermodynamics (How?)

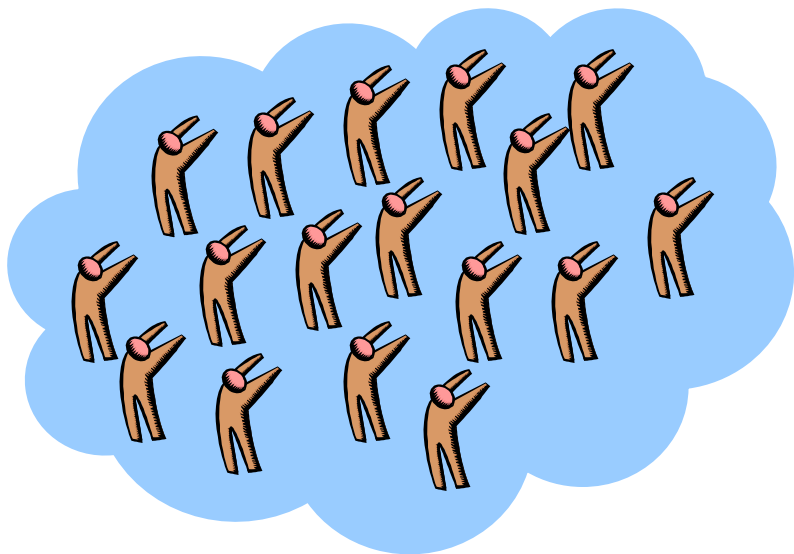


Micro-scale: statistical physics (Why?)

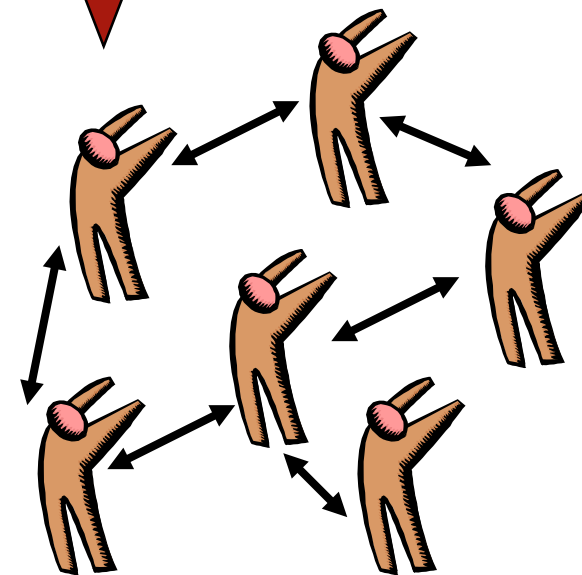




Why social systems?



Sociology
Fundamental unit:
a social group (macro-scale)



Social Psychology
Fundamental unit:
a person (micro- scale)





Inspiration and hints
from statistical physics



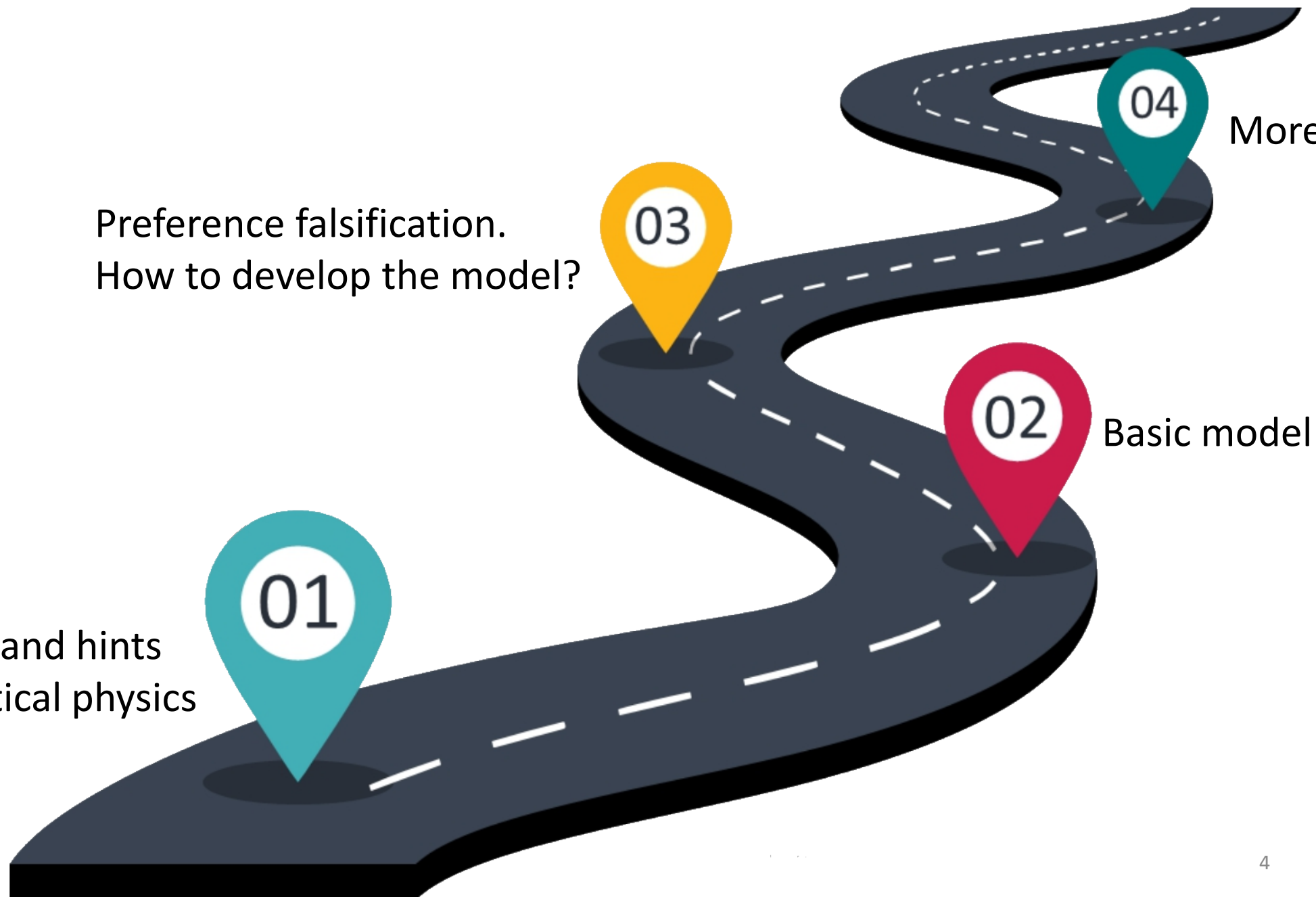
Preference falsification.
How to develop the model?



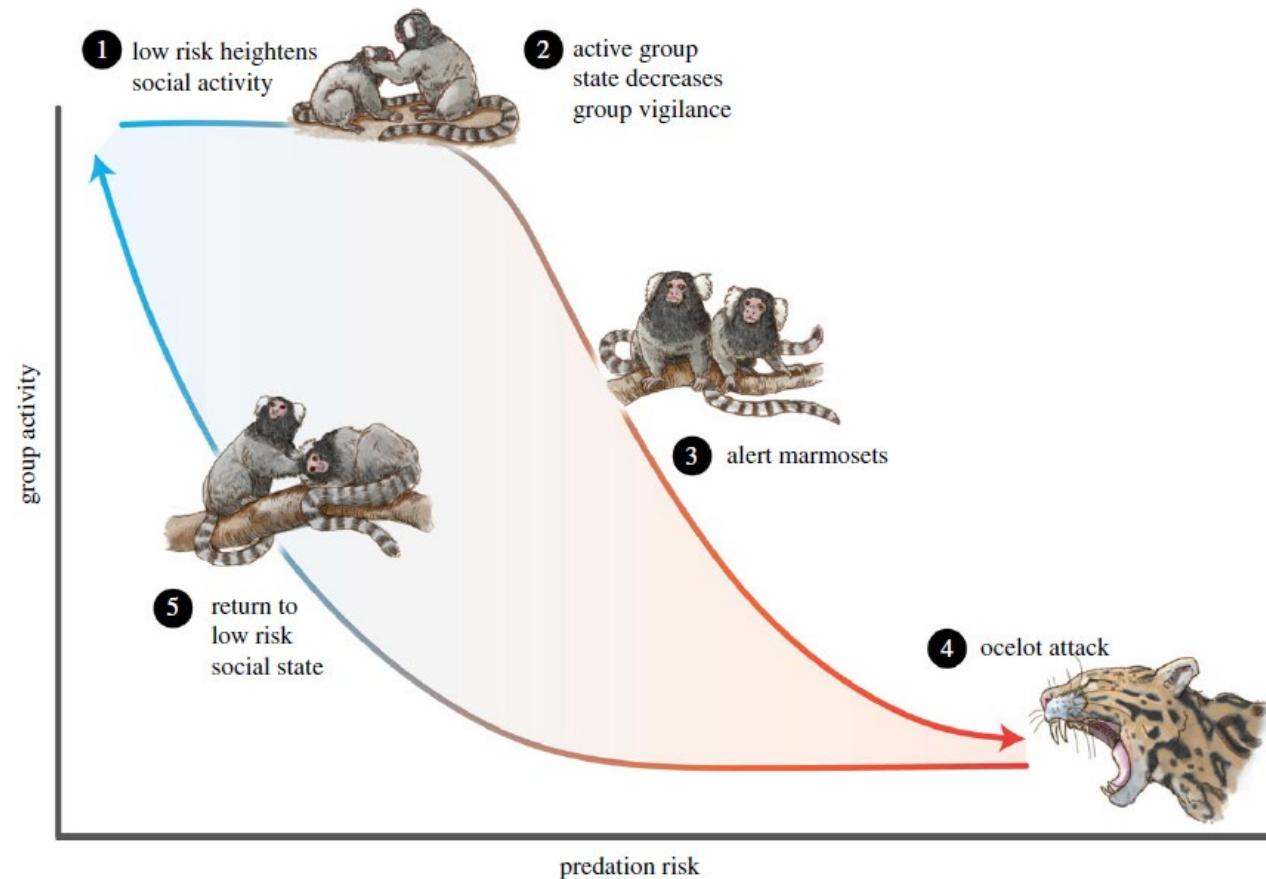
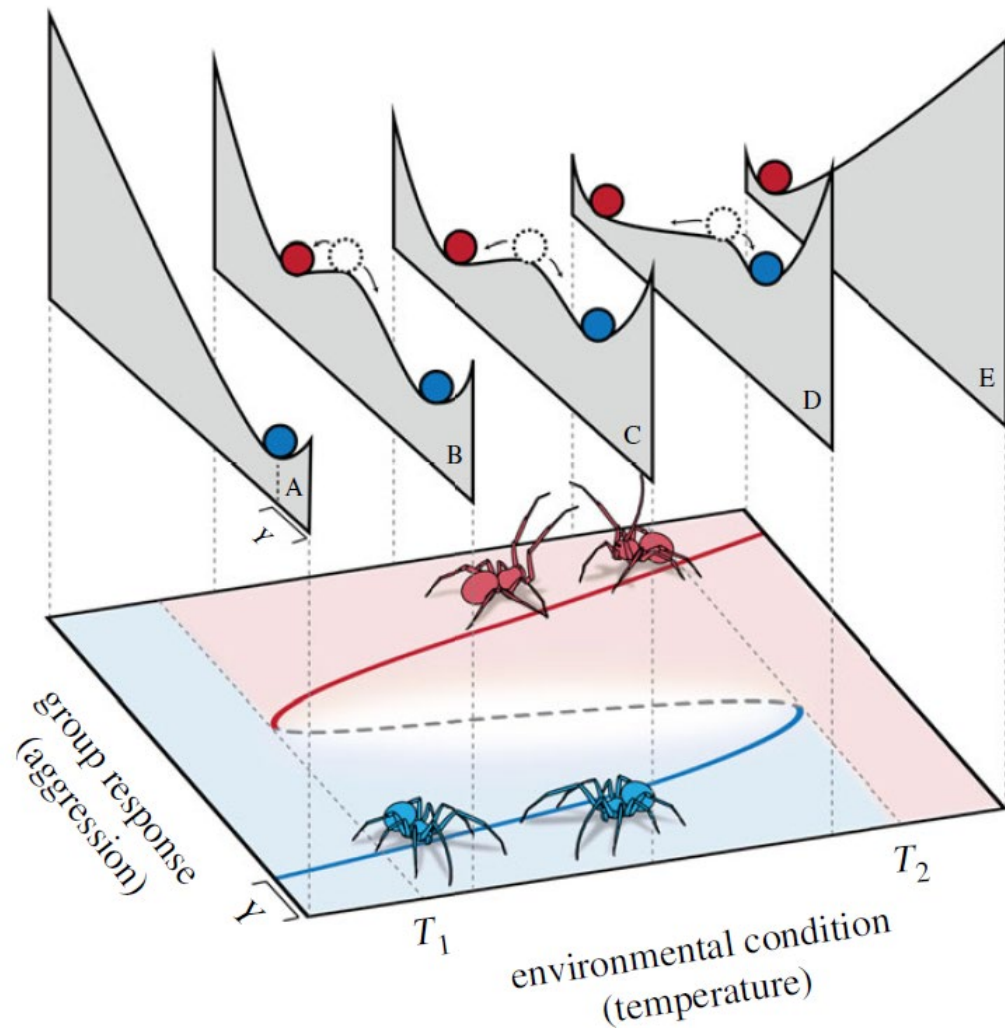
Basic model



More?



Inspiration



Doering GN et al. 2018 Social tipping points in animal societies in response to heat stress: timing, recovery and hysteresis. *Nature Ecol. Evol.* 2, 1298–1305.

Pruitt JN et al. 2018 Social tipping points in animal societies. *Proc. R. Soc. B* 285: 20181282



The new inspiration



Covid-19 Comment Pieces

Hysteresis and the sociological perspective in a time of crisis

Hannah Graham 

University of Stirling, UK

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2020, Vol. 63(4) 450–452
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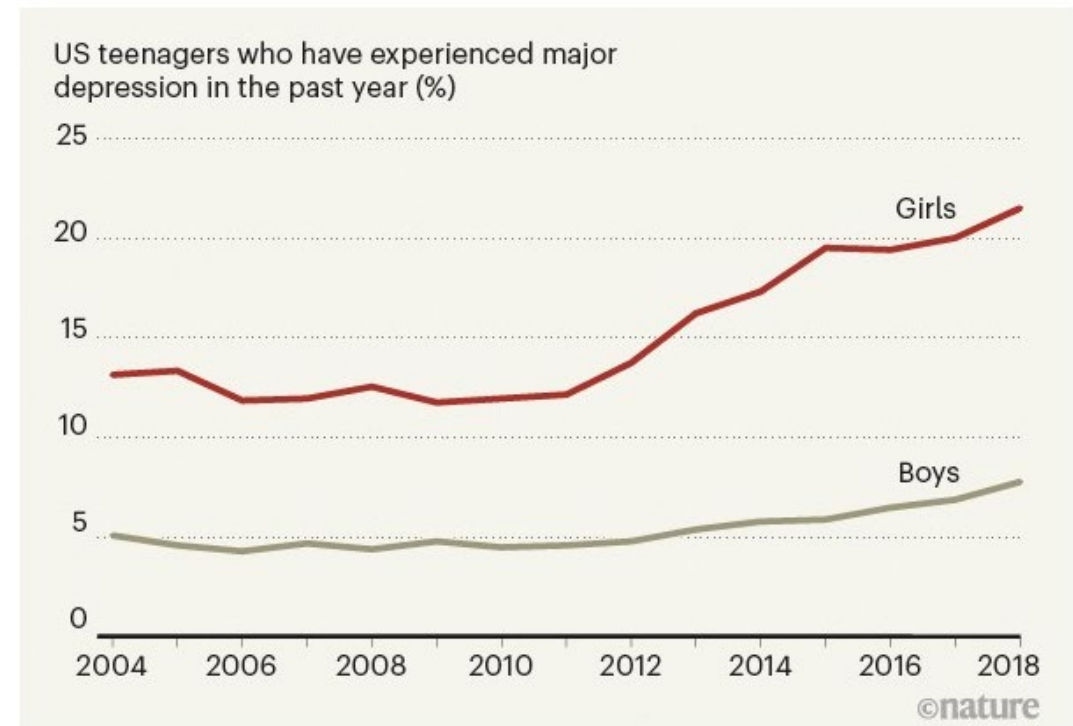
Abstract

Hysteresis is a versatile concept for volatile times. Pierre Bourdieu's sociological use recognises hysteresis in times of dislocation and disruption between field and habitus, 'in particular, when a field undergoes a major crisis and its regularities (even its rules) are profoundly changed' (Bourdieu, 2000: 160). In considering the issues and implications of the COVID-19



Questions

- Why achieving a desirable rate of vaccination in modern societies is challenging?
- What can be the reason of the rise of mental health disorders among undergraduate students?
- Why some ideas, practices and products spread so fast while the diffusion of other is hampered?





Hysteresis – what is it?

- The dependence of the state of a system on its history
- The term is derived from ὑστέρησις = lagging behind (Ancient Greek)
- It was coined ~ 1890 by Sir James Alfred Ewing to describe the behavior of magnetic materials
- Everyday example?
- What it has to do with social systems?

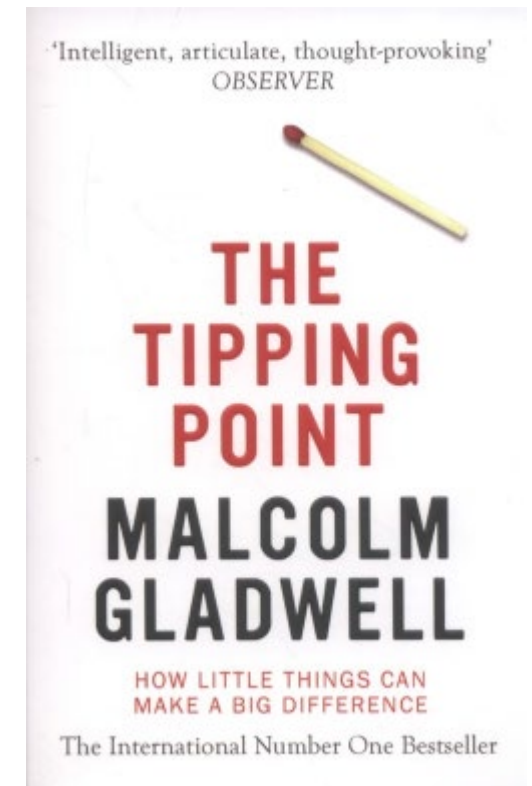


Hysteresis & tipping points in public opinion

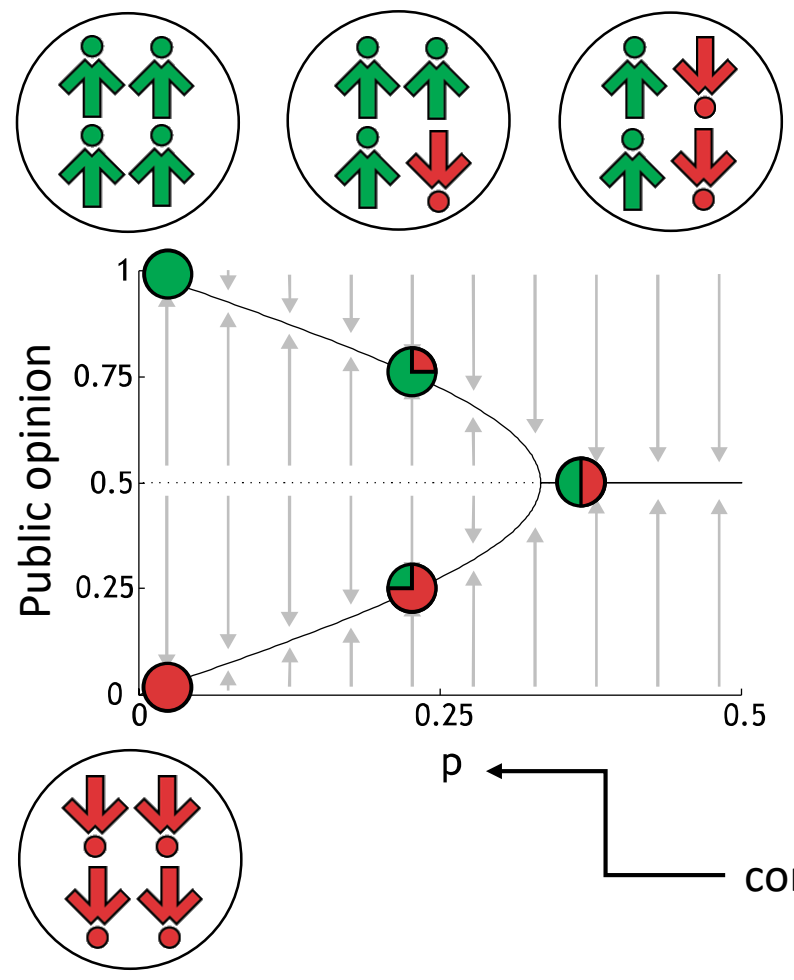
- Empirical studies suggest that:
 - it remains seemingly resistant to change (hysteresis)
 - sudden, abrupt shift of opinion can be observed at the tipping point

[1] M. Scheffer, et al, Slow response of societies to new problems: Causes and costs. *Ecosystems* (2003)

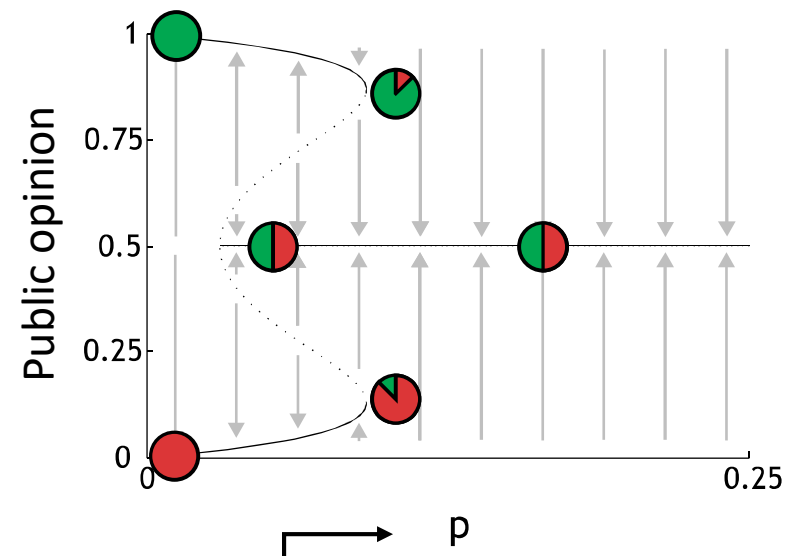
[2] D. Centola, et al. Experimental evidence for tipping points in social convention. *Science* (2018)



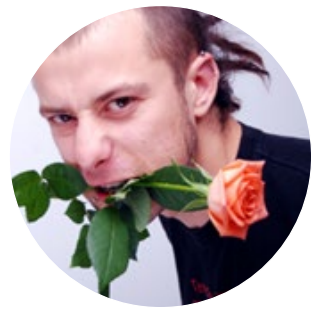
Hints from the theory of phase transitions



Continuous phase transitions

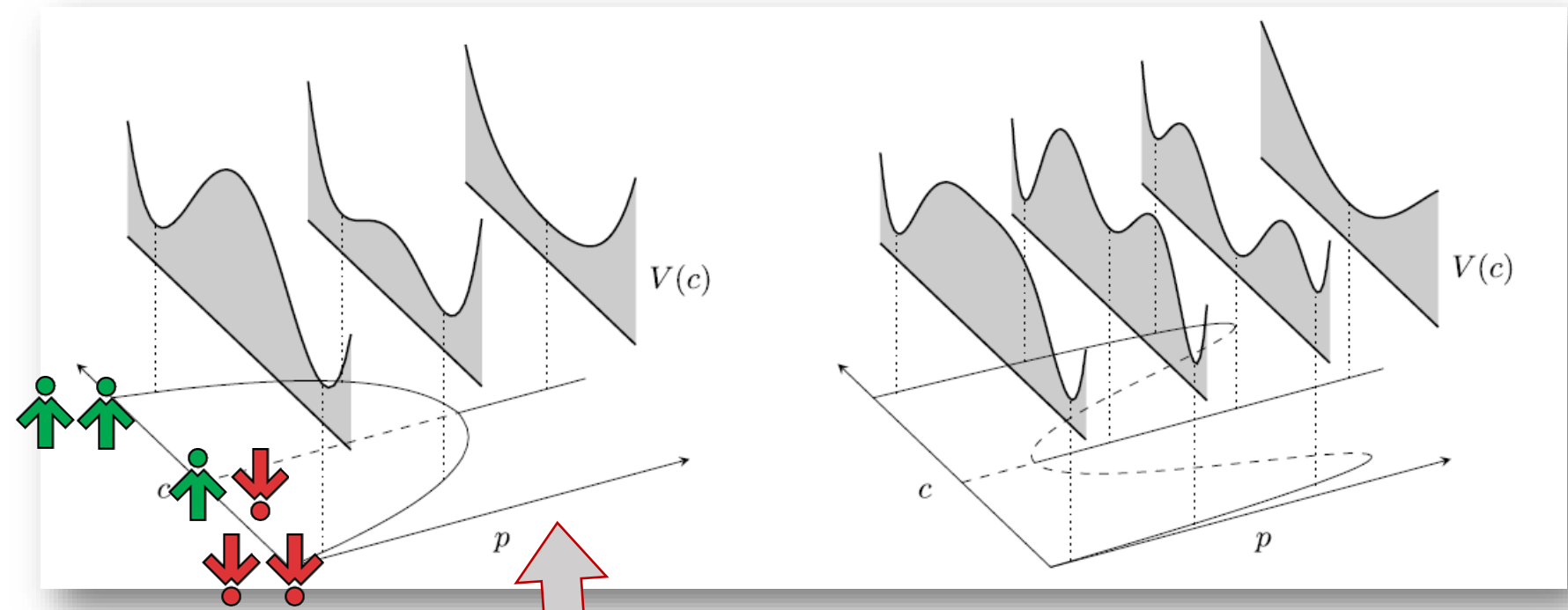


Discontinuous phase transitions



(c) 2013 Piotr Nyczka

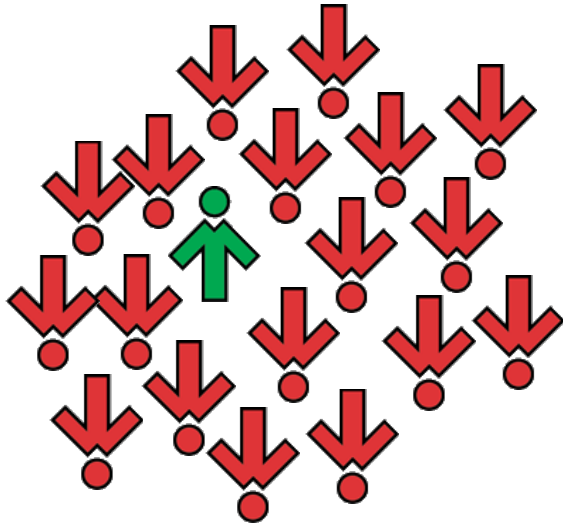
Hints from the theory of phase transitions



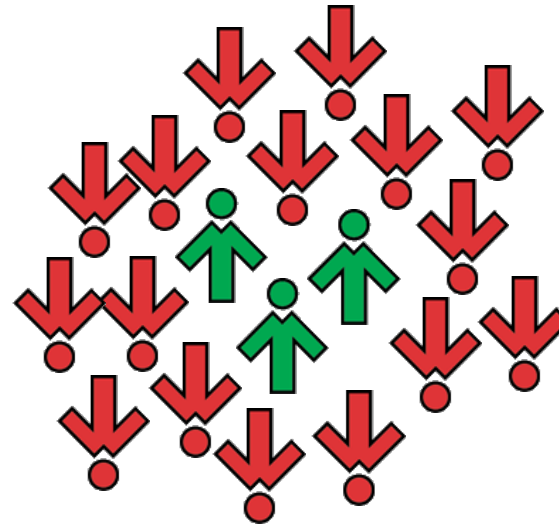
(c) 2019 Arek Jędrzejewski

	CONTINUOUS	DISCONTINUOUS
Metastability, hysteresis, phase coexistence		✓
Latent heat (energy)		✓
critical nucleus (seed)		✓

Hysteresis and critical nuclei (critical mass)



Not enough



Critical mass

[1] D. Centola, etal. Experimental evidence for tipping points in social convention. Science (2018)



What factors hamper
and what factors promote
social hysteresis?



Preference falsification.
How to develop the model?

Inspiration and hints
from statistical physics

01

03

02

04

Basic model

More?



The answer within agent-based models

- Expected: inertia (memory) on the individual level supports social hysteresis
- Surprise: memoryless agents can also „produce” hysteresis (memory of the system)
 - The size of the group of influence
 - ...

How to investigate it?

Example: the q-voter model



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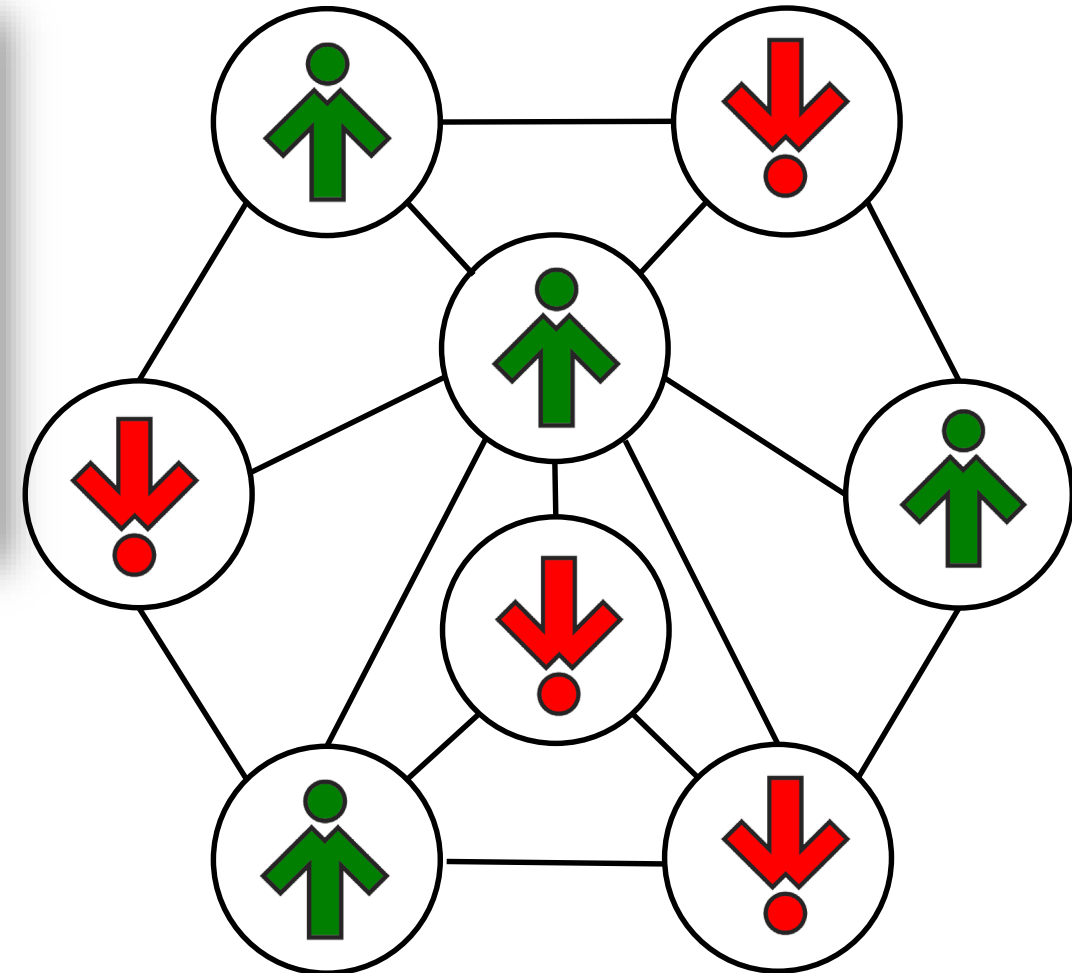
From statistical physics to social sciences / *De la physique statistique aux sciences sociales*

Statistical Physics Of Opinion Formation: Is it a SPOOF?

Physique statistique de la formation d'opinion : est-ce une blague ?

Arkadiusz Jędrzejewski*, Katarzyna Sznajd-Weron

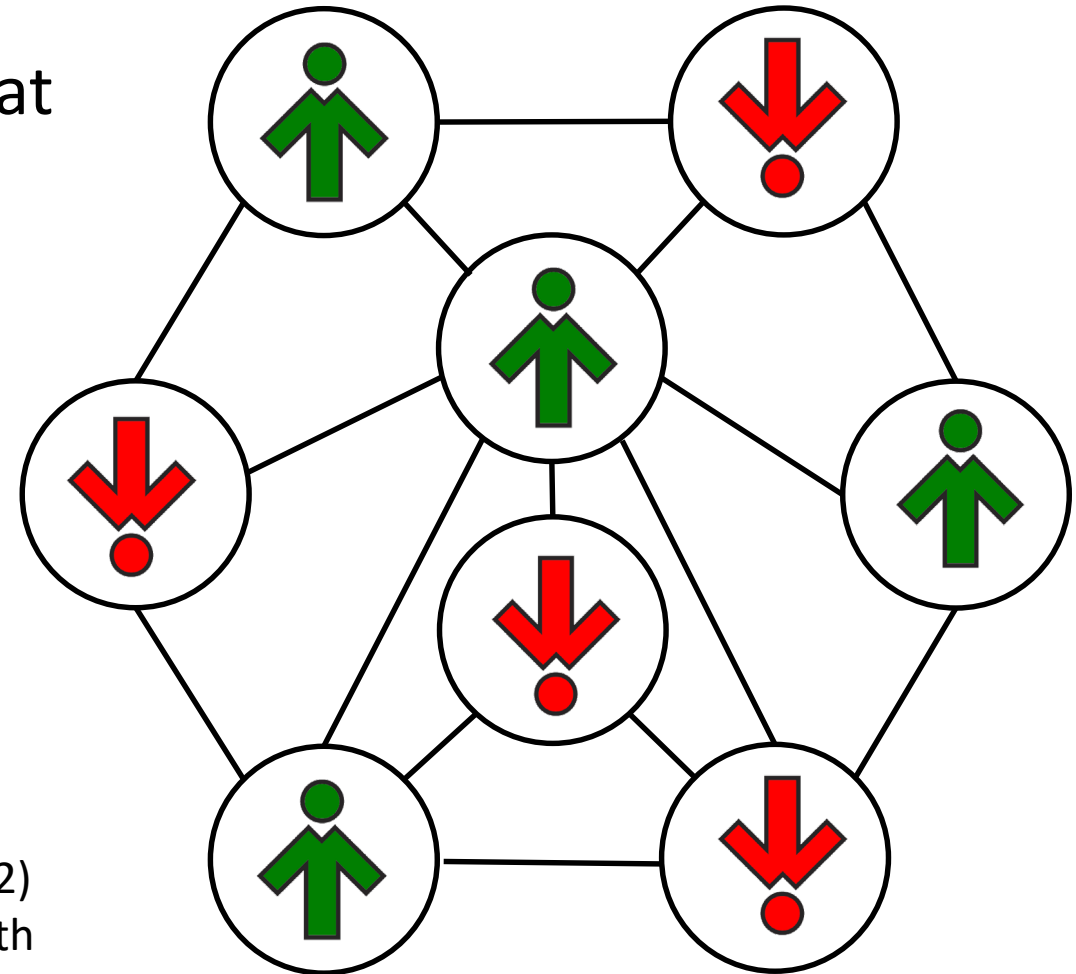
Department of Theoretical Physics, Faculty of Fundamental Problems of Technology, Wrocław University of Science and Technology, Wrocław, Poland



$$S_i(t) = \pm 1 \quad \text{Individual opinion}$$

The q -voter model with independence

1. Choose one voter at random, located at site i



- [1] P. Nyczka, K. Sznajd-Weron, and J. Cislo, *Phase transitions in the q -voter model with two types of stochastic driving*, PRE (2012)
[2] A. Jędrzejewski, *Pair approximation for the q -voter model with independence on complex networks*, PRE (2017)

The q-voter model with independence

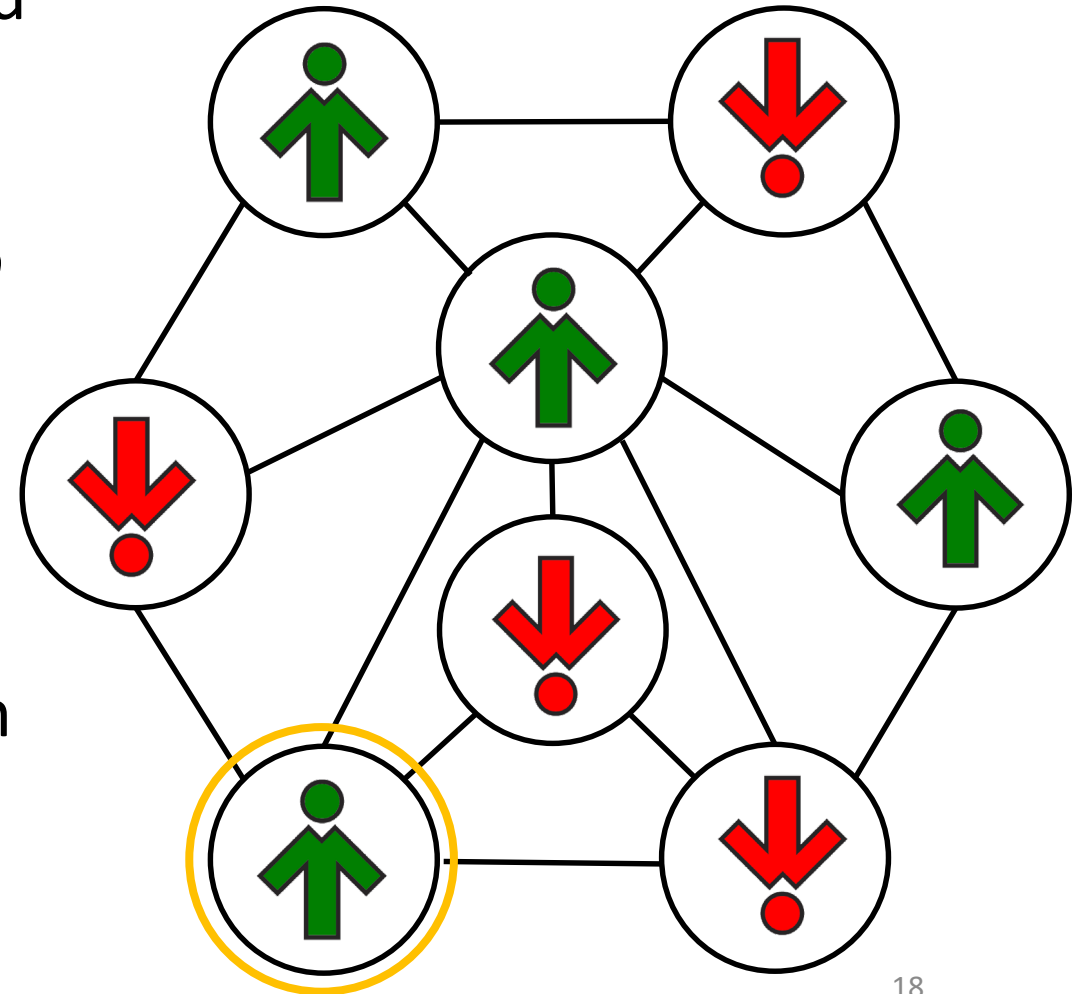
1. Choose one voter at random, located at site i
2. Update the opinion S_i
 - i. Independence with probability p
→ change the opinion to the opposite one with probability f

How?

1. Choose $r \sim U(0,1)$, if $r < p$ then independence

Example 1:

$$r = 0.1, p = 0.2, f = 1$$



The q-voter model with independence

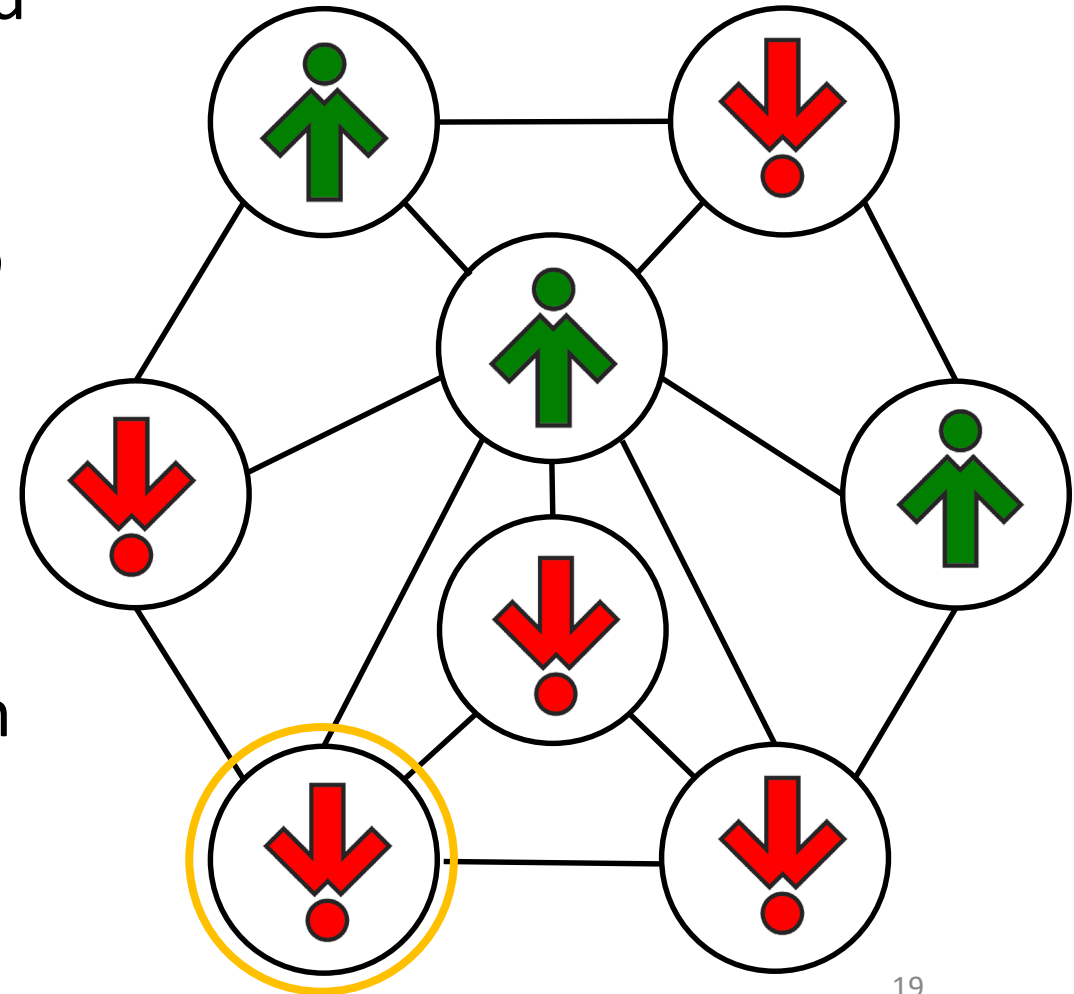
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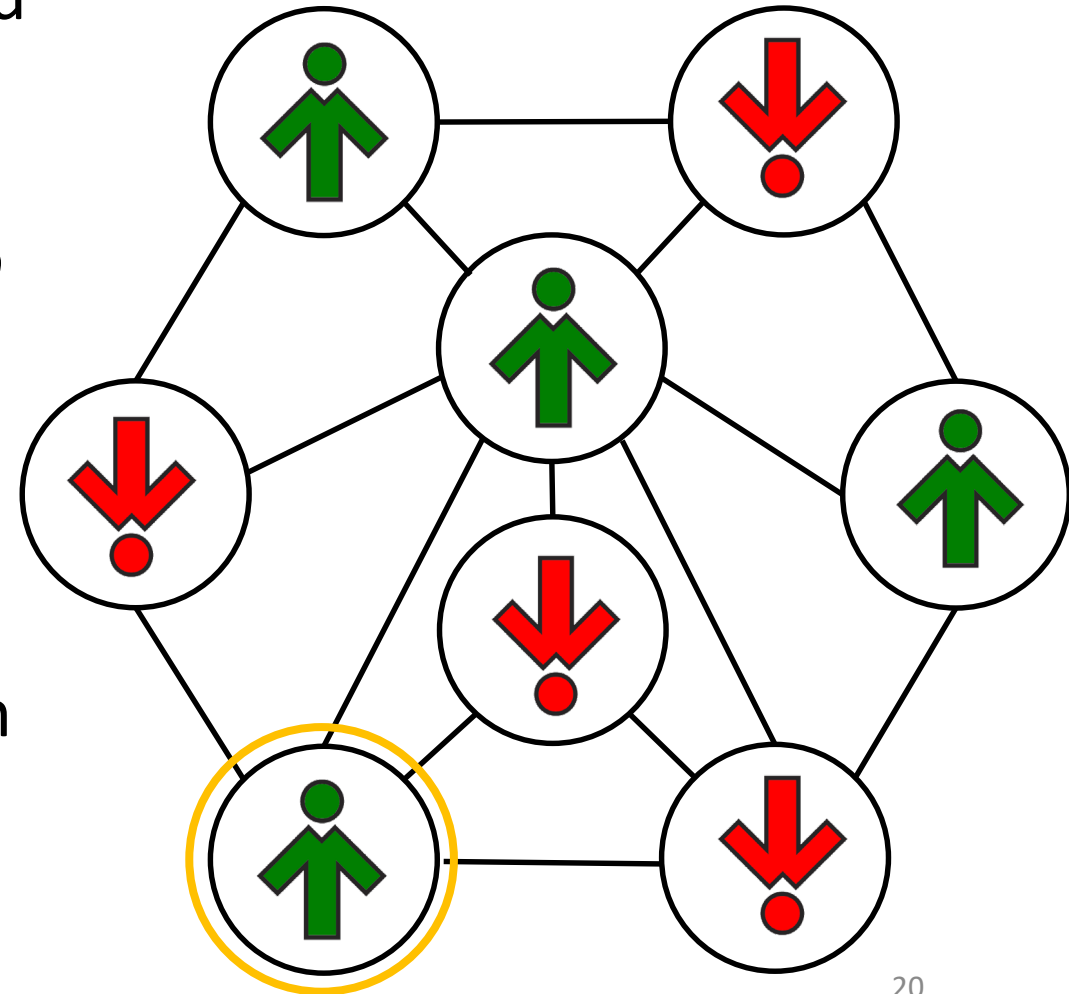
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Example 2:

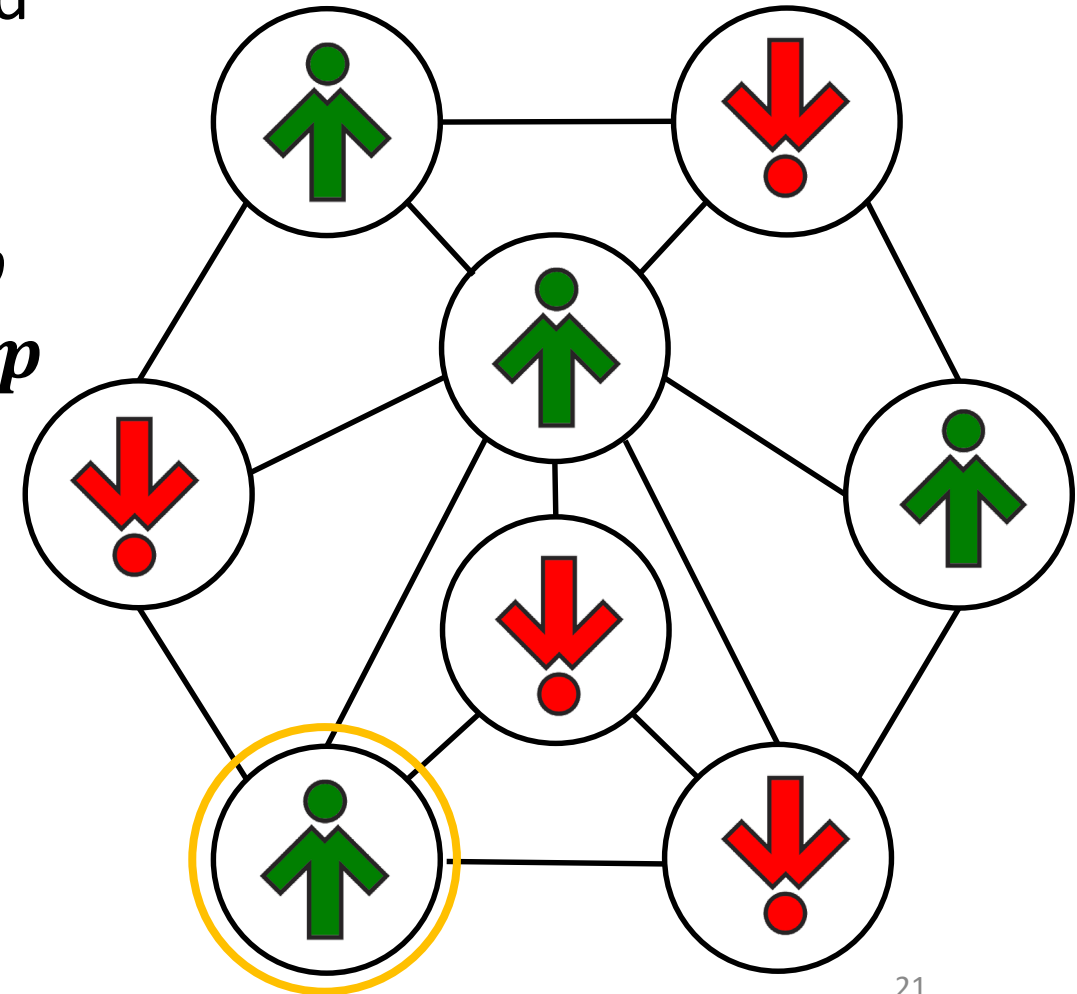
$$r = 0.3, p = 0.2, f = 1$$



The q -voter model with independence

1. Choose one voter at random, located at site i
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 - ii. Conformity with probability $1 - p$

→ pick randomly q neighbors without repetition

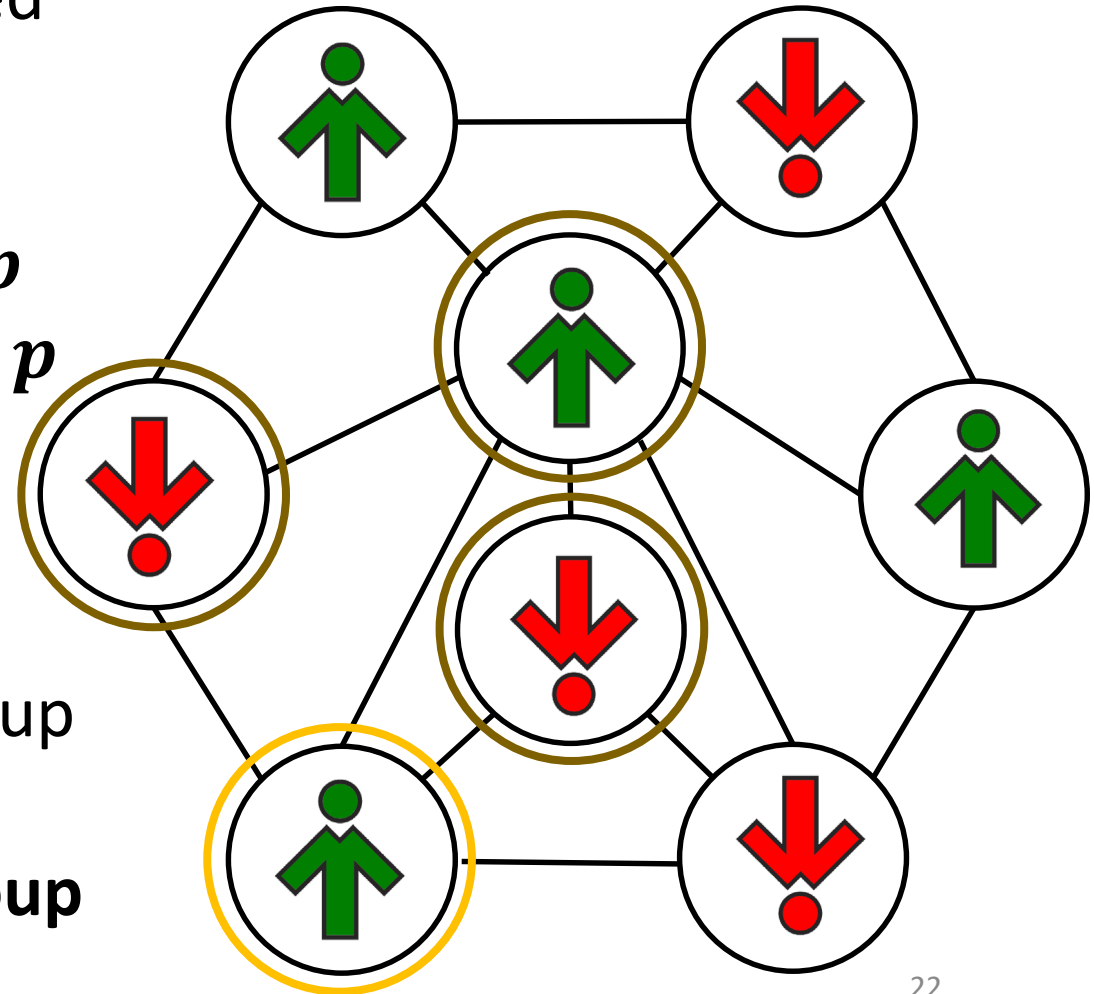


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1. Choose one voter at random, located at site i
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 - ii. Conformity with probability $1 - p$
→ pick randomly q neighbours without repetition,

unanimous group - S_i adjust the group

Example 3: $q=3$, not unanimous group of influence – nothing changes

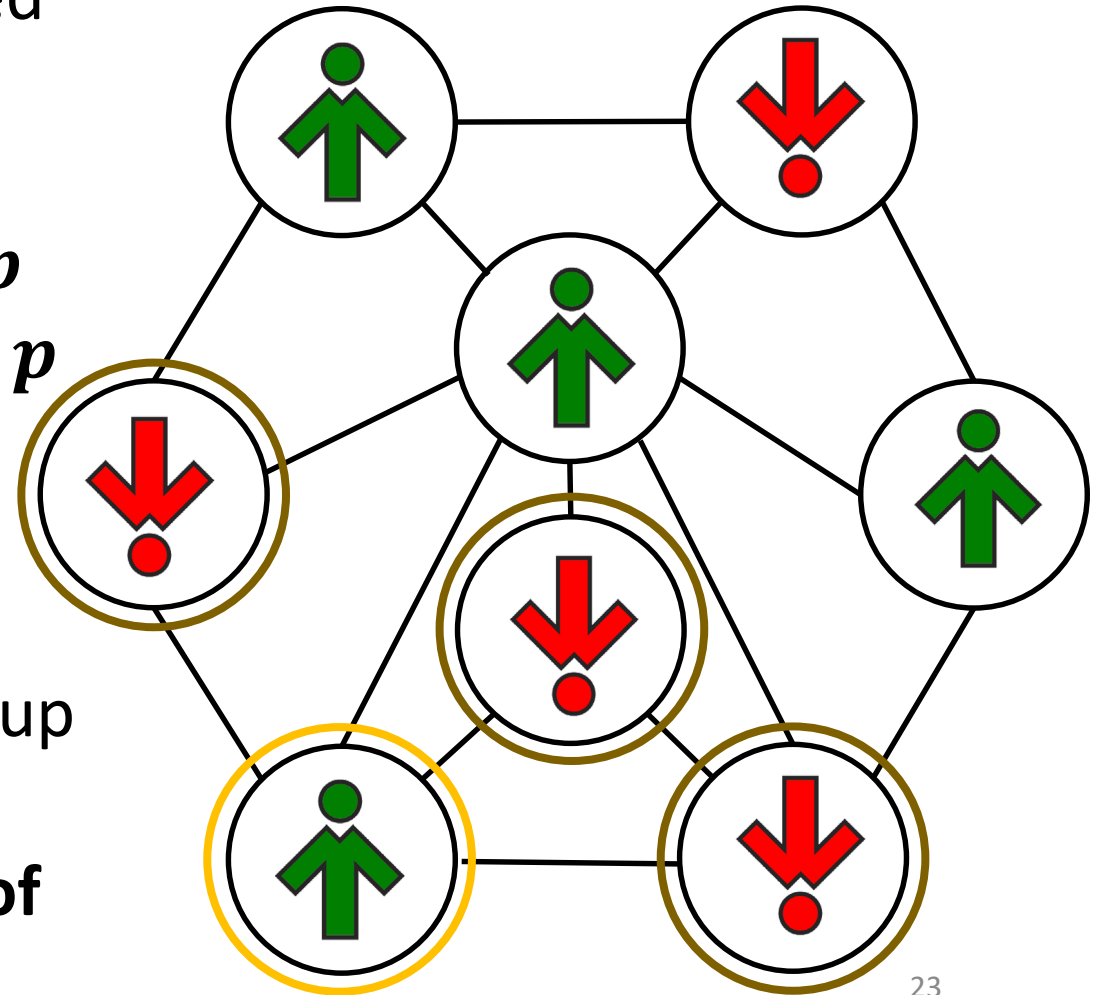


The q -voter model with independence

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 \rightarrow pick randomly q neighbours without repetition,

unanimous group - S_i adjust the group

Example 4: $q=3$, unanimous group of influence – adjust

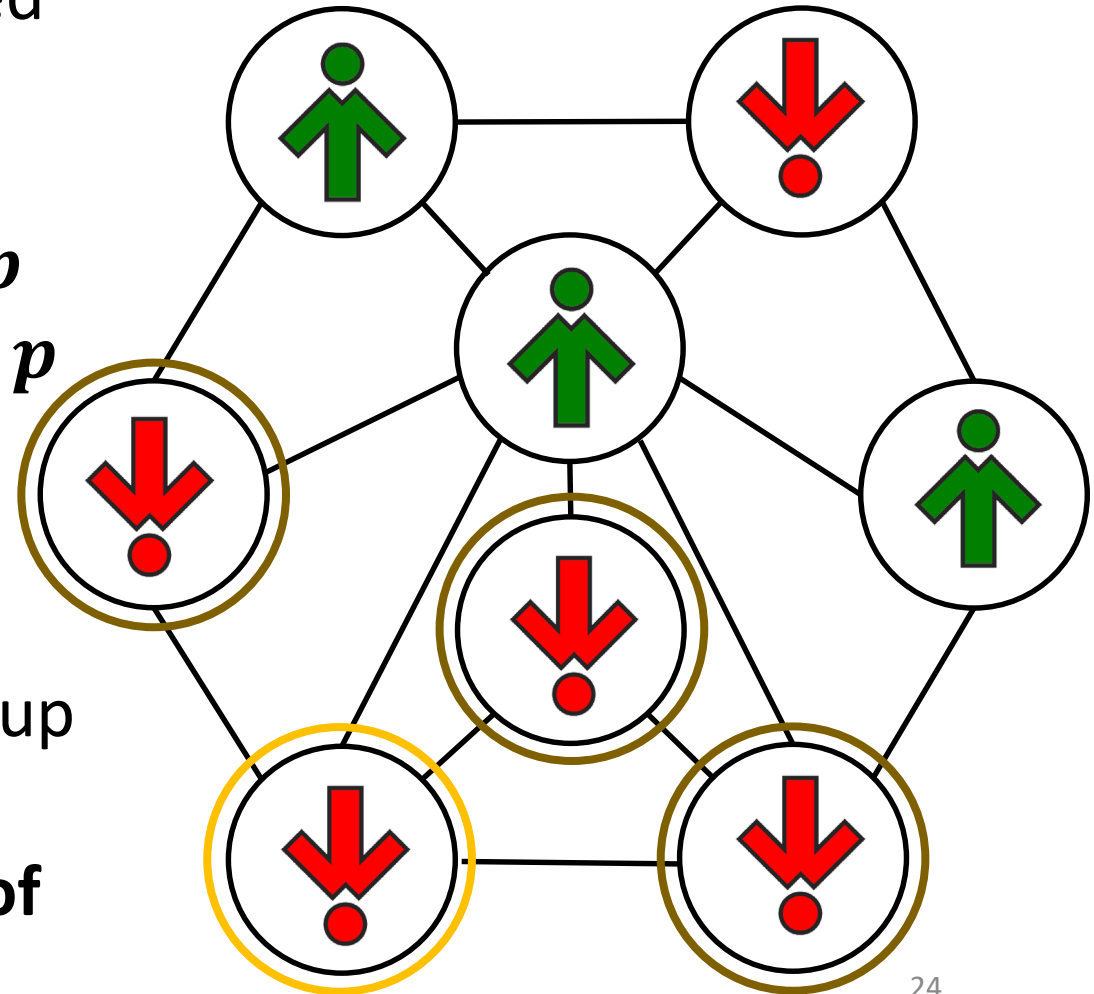


The q -voter model with independence

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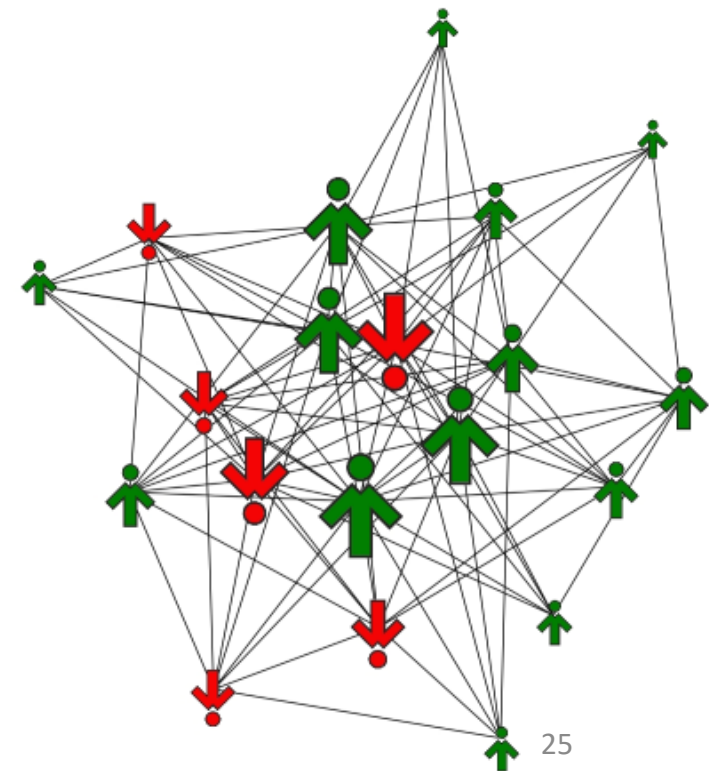
unanimous group - S_i adjust the group

Example 4: $q=3$, unanimous group of influence – adjust



Variables, parameters and measurements

- N – the number of agents (constant)
- $S_i(t) = \pm 1$, $i = 1, \dots, N$ individual opinion (dynamical variable)
- t – time measured in Monte Carlo Steps (MCS)
- 1 MCS = N updates
- Model's parameters:
 - q (size of the influence group), p (independence)
 - f (flexibility, usually $f = 1/2$)
- What do we measure?
 - $m(t) = \frac{1}{N} \sum_i S_i(t) = \frac{N_+(t) - N_-(t)}{N}$
 - $c(t) = \frac{N_+(t)}{N}$, $m(t) = 2c(t) - 1$

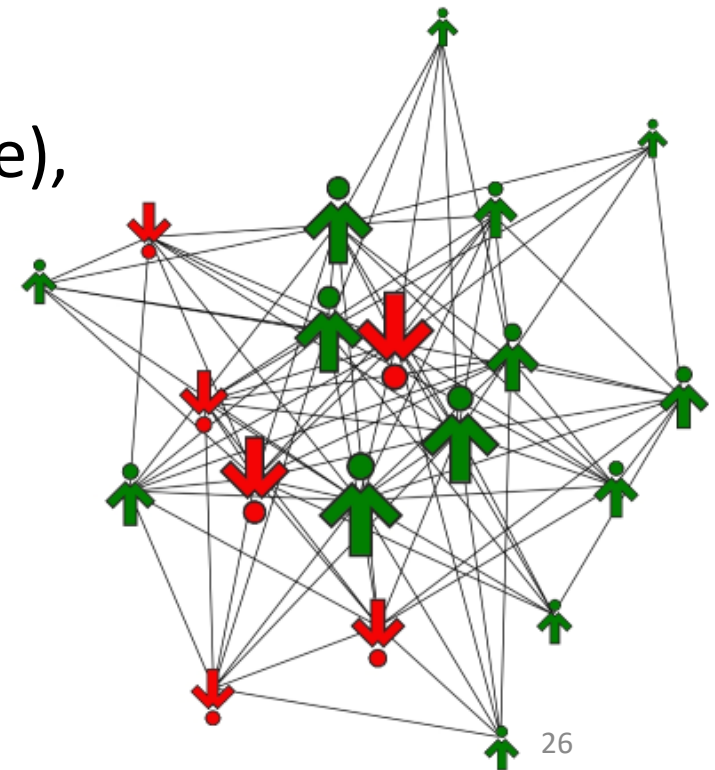


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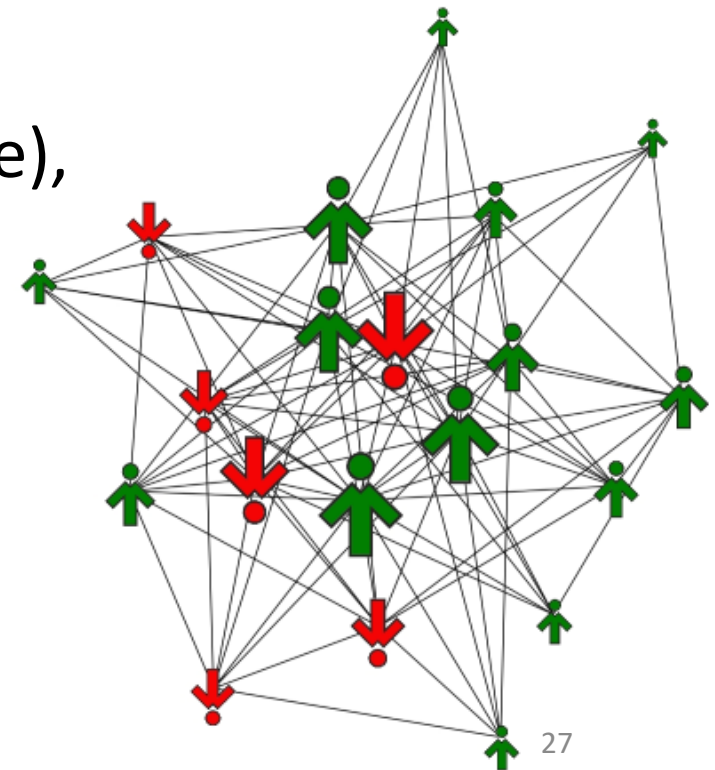
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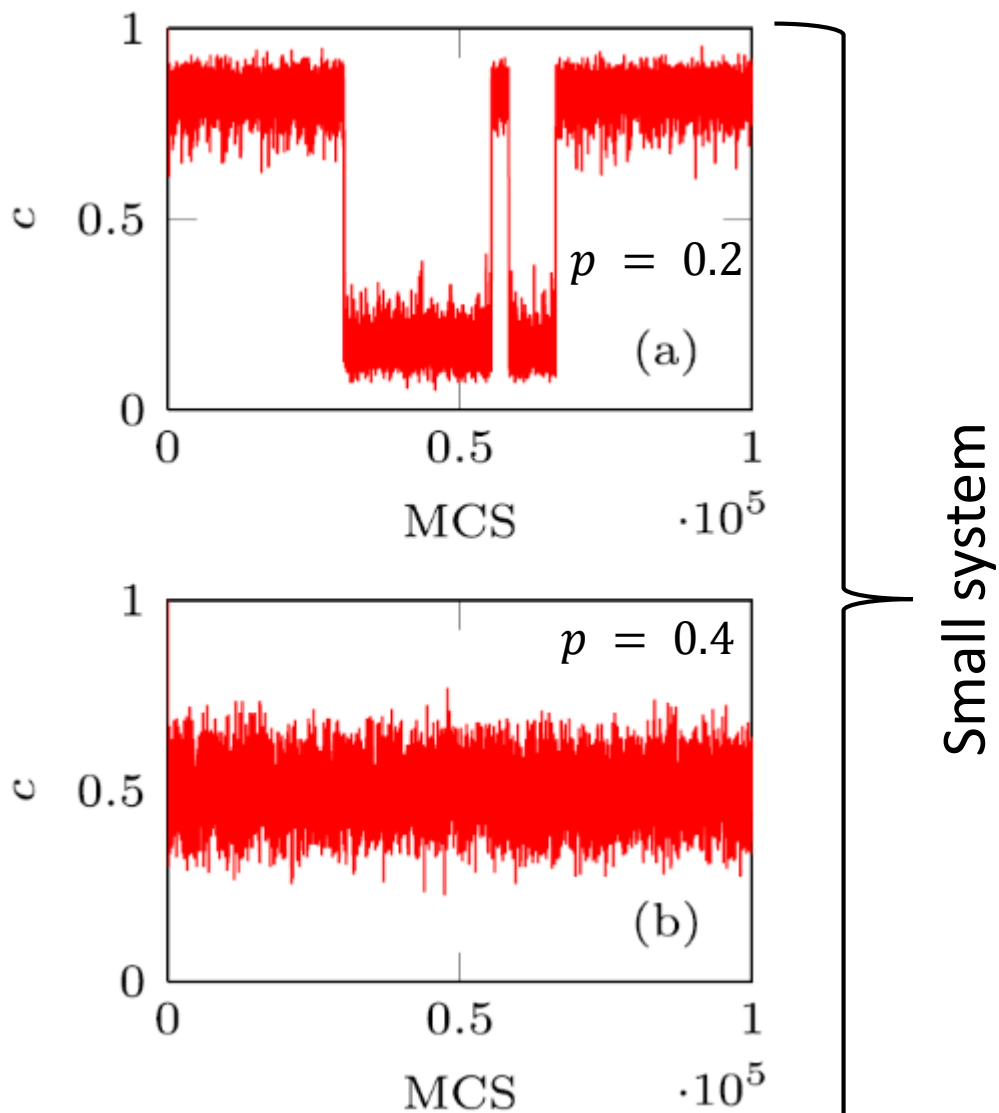
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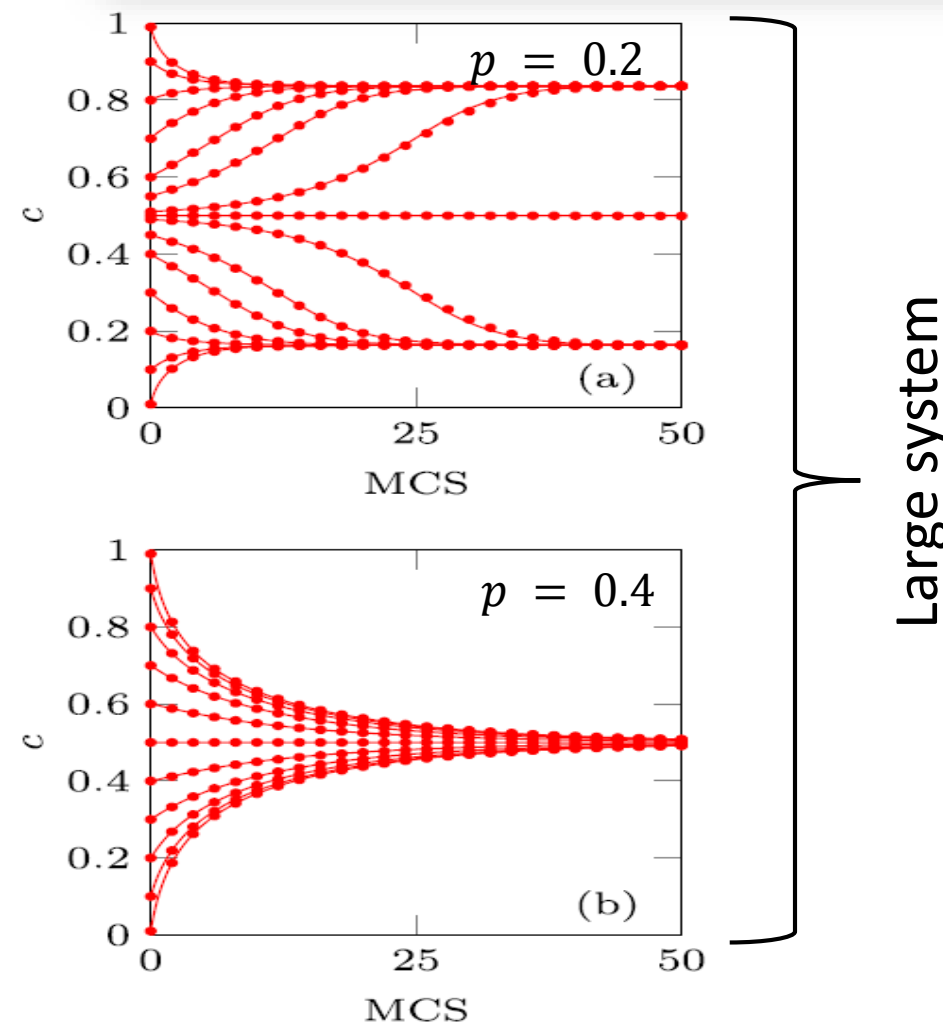




Sample trajectories

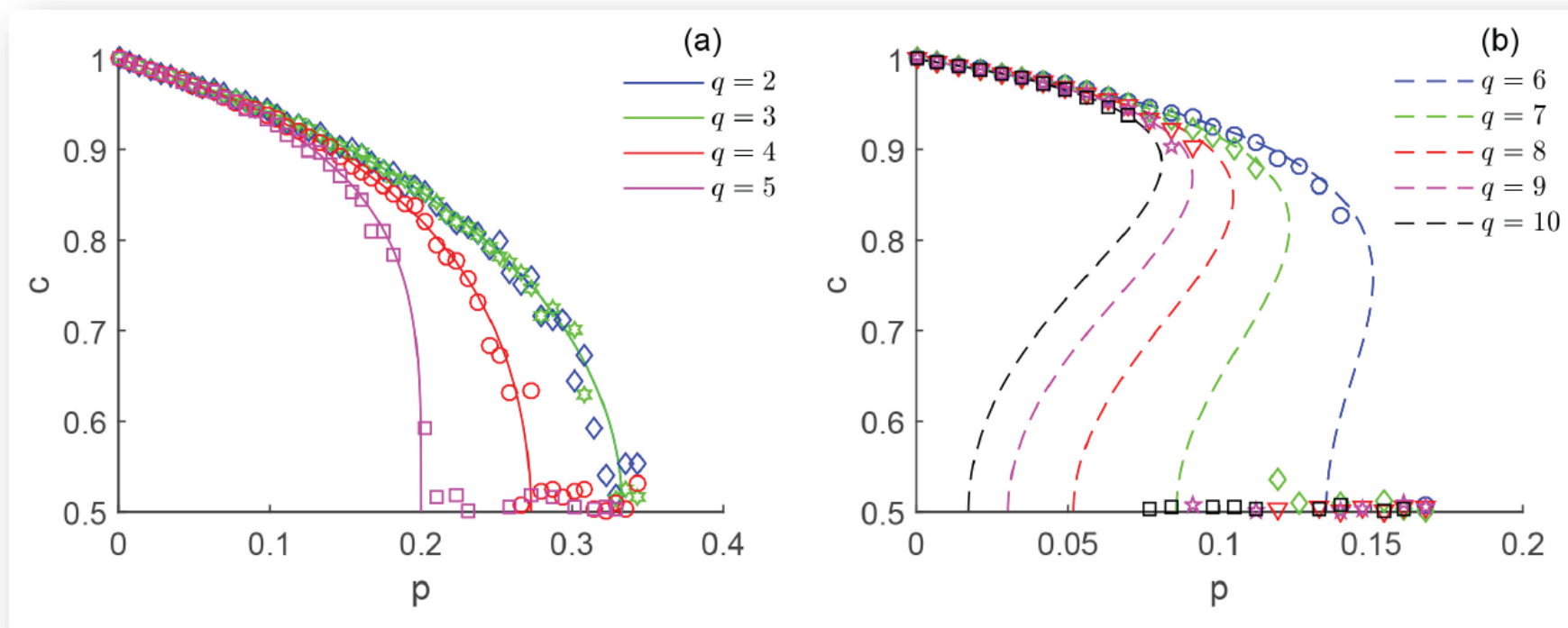


Example: $q = 2, N = 200, c(0) = 1$



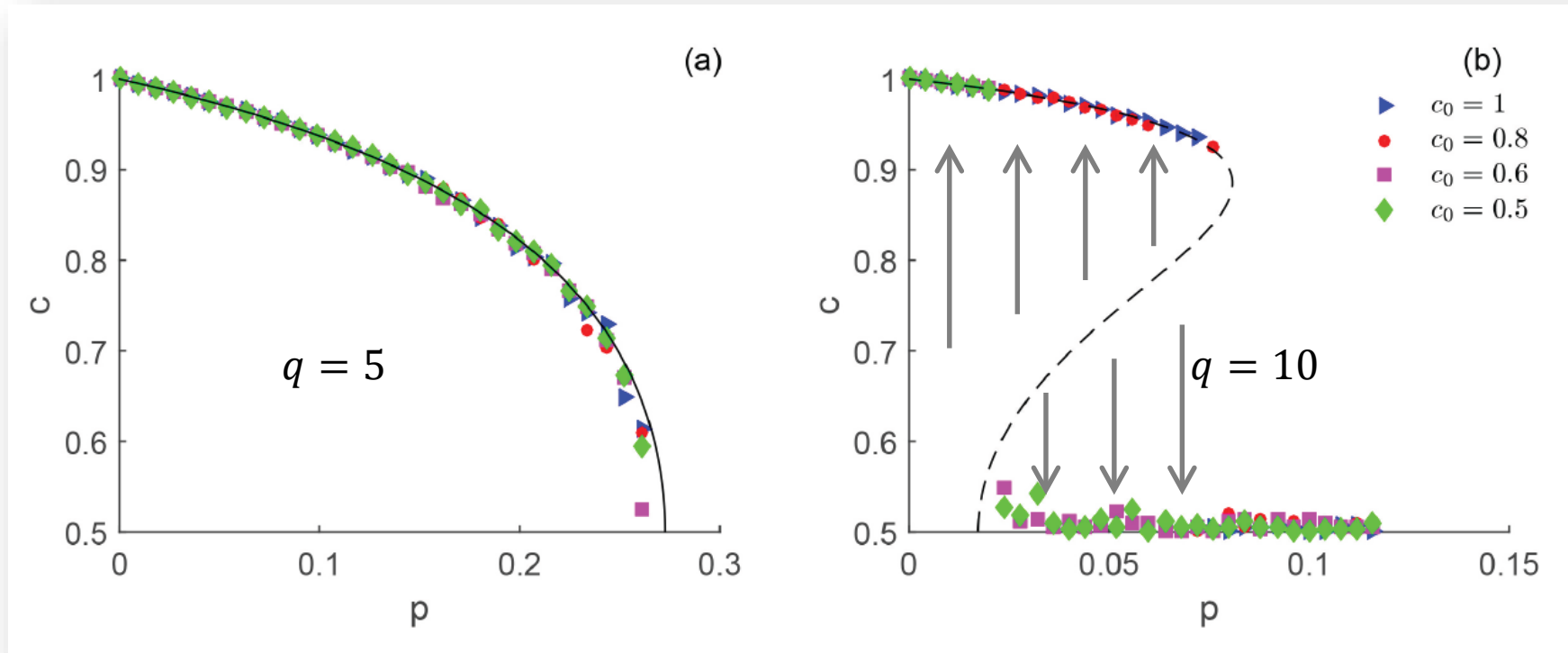
Example: $q = 2, N = 10^5, c(0) = c_0 \in (0,1)$

Stationary states – „small simulations”



- Simulations on the complete graph, $N = 1000$
- Measurement after 1000 *MCS*
- Average over the small number of samples
- Simulations by Urszula Grochocińska (3rd year student)

How the hysteresis can be seen?



- Simulations on the complete graph, $N = 1000$
- Measurement after 1000 *MCS*
- Average over the small number of samples
- Simulations by Urszula Grochocińska (3rd year student)



How to develop the model?
Preference falsification.

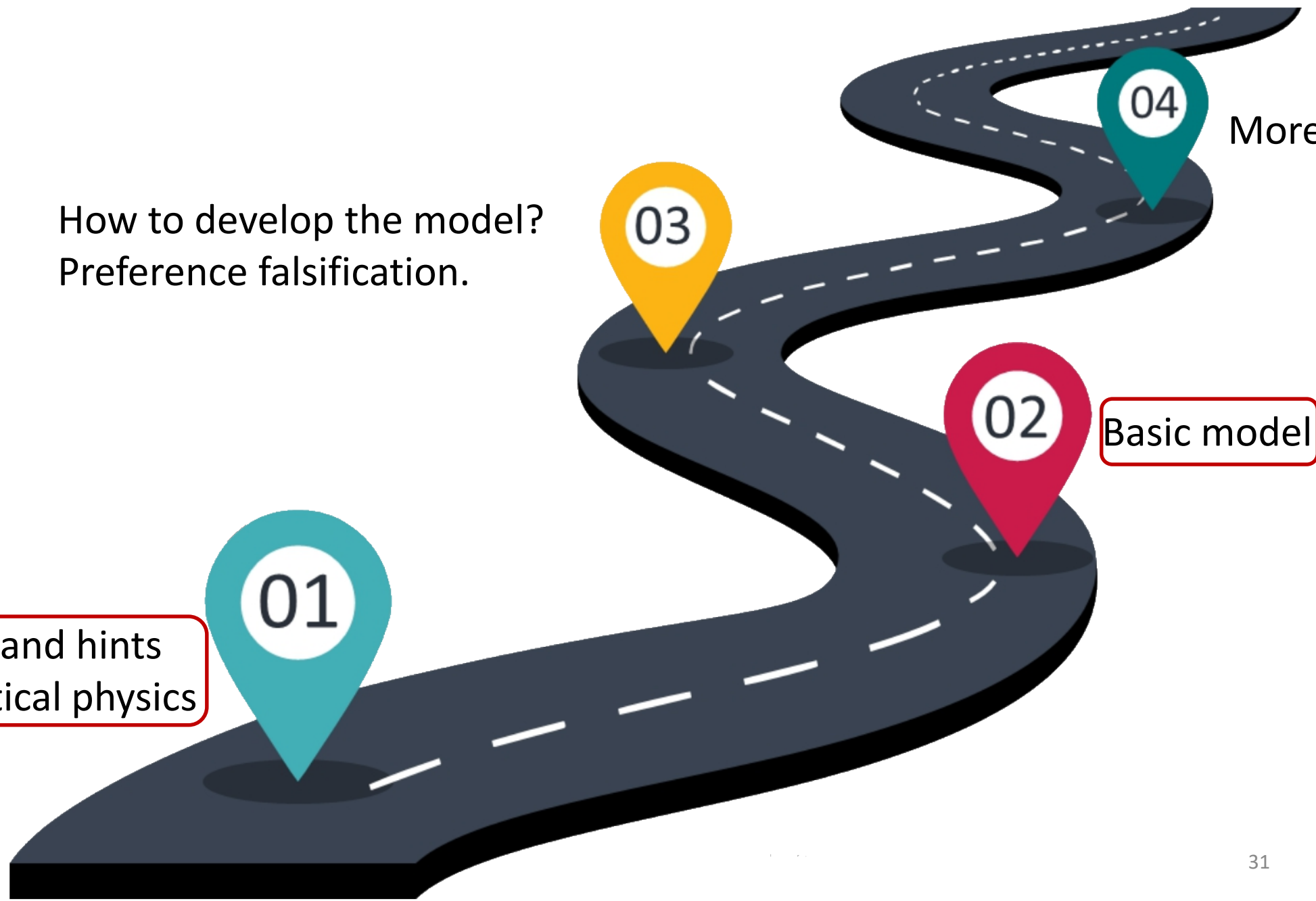
Inspiration and hints
from statistical physics



Basic model

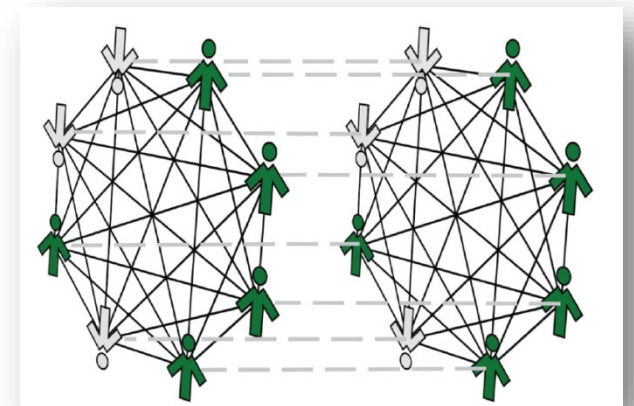


More?



The answer within agent-based models

- Expected: inertia (memory) on the individual level supports social hysteresis
- Surprise: memoryless agents can also „produce” hysteresis (memory of the system)
 - The size of the group of influence
 - The number of layers in the social network
 - Hidden preferences



PHYSICAL REVIEW E **92**, 052812 (2015)

Phase transitions in the q -voter model with noise on a duplex clique

Anna Chmiel and Katarzyna Sznajd-Weron

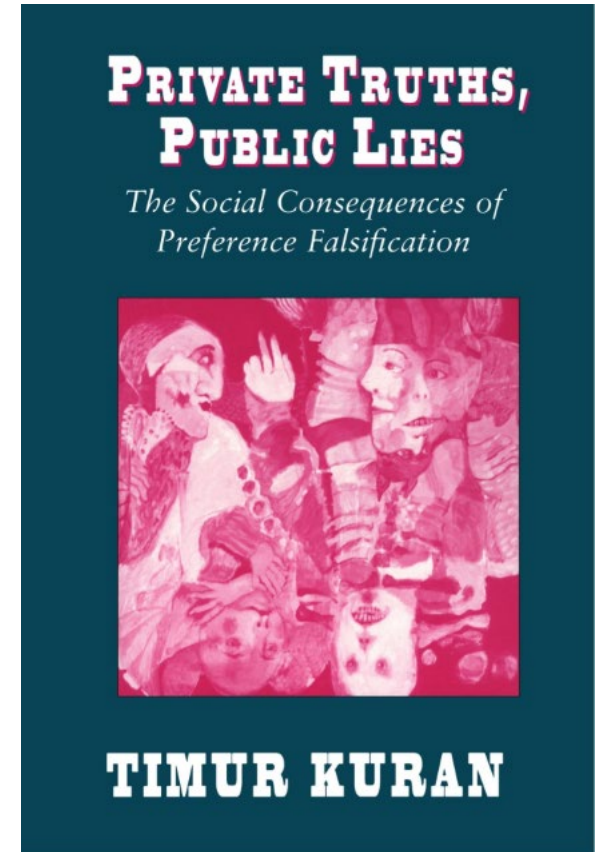
Department of Theoretical Physics, Wrocław University of Technology, Wrocław, Poland

(Received 11 March 2015; revised manuscript received 20 July 2015; published 24 November 2015)

Each node possesses a counterpart node in the second level³²

Preference falsification

- The act of communicating a preference that differs from one's true preference
- Main reason: believe the expressed preference is more acceptable socially
- Huge social and political consequences, ex: unanticipated revolutions
- Opinion on two levels: public and private
- Not like in the CODA model:
 - André C.R. Martins, Continuous opinions and discrete actions in opinion dynamics problems, IJMPC 19 (2008)

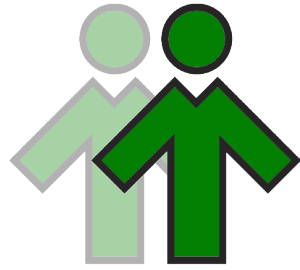
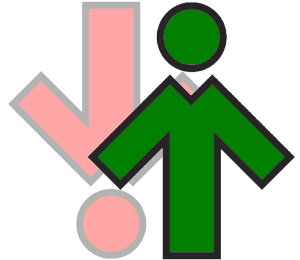
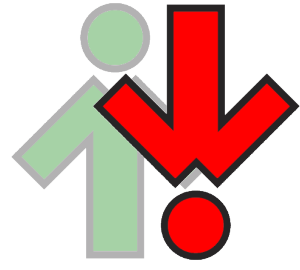
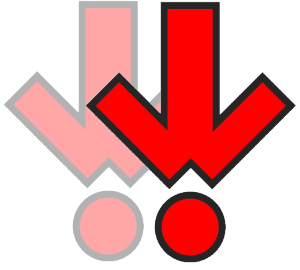


A new work of art in your boss's home



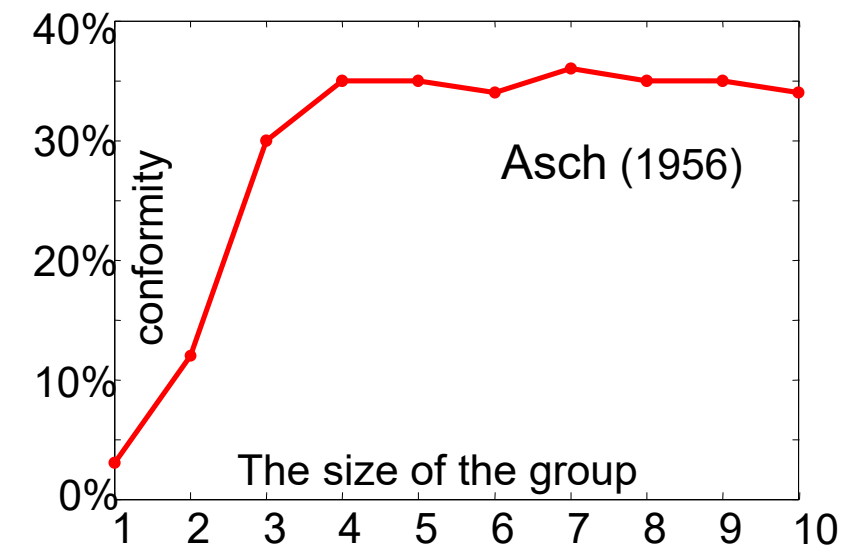
The model

- N agents
 - Public opinion $S_i(t) = \pm 1$
 - Private opinion $\sigma_i(t) = \pm 1$
- Only $S_i(t)$ is seen by others
- Two types of social responses
 - Independence with p
 - Conformity with $1 - p$
- Conformity
 - compliance (unanimous q -panel)
 - disinhibitory contagion

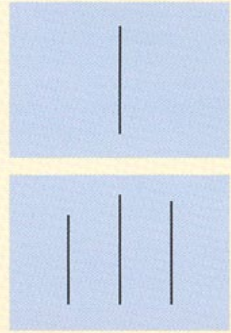
		PRIVATE (INTERNAL)	
		$\sigma_i(t) = +1$	$\sigma_i(t) = -1$
PUBLIC (EXTERNAL)	$S_i(t) = +1$		
	$S_i(t) = -1$		


Two types of conformity

- Compliance: public conformity without private acceptance
 - Asch experiment
 - Increases with the size of the group
 - Unanimity is crucial

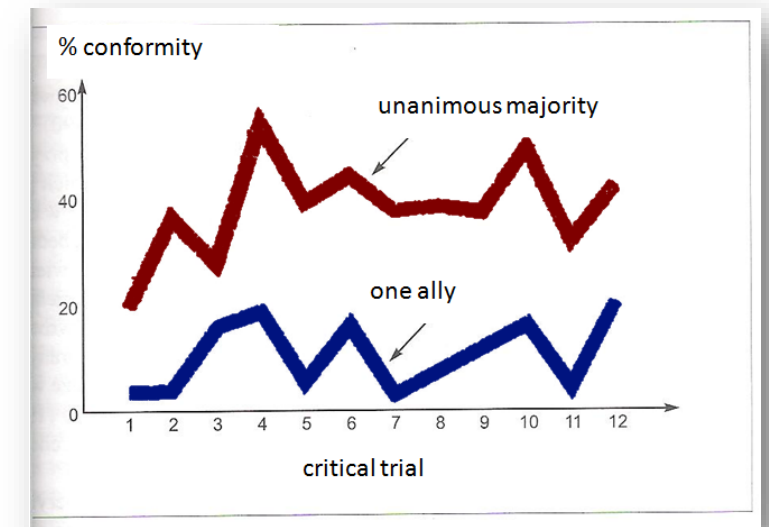


Asch's Classic Study of Conformity





Subjects were shown the cards at the left and asked to choose the line in the picture on the bottom that was the same length as the line in the picture on the top. The confederates deliberately chose incorrect answers to see if the unsuspecting subject (#6) would go along with the majority.



Two types of conformity

- Disinhibitory contagion
 - Appears in the case of the internal intra-psychic conflict
 - Single person can influence



RESEARCH ARTICLE

Think then act or act then think?

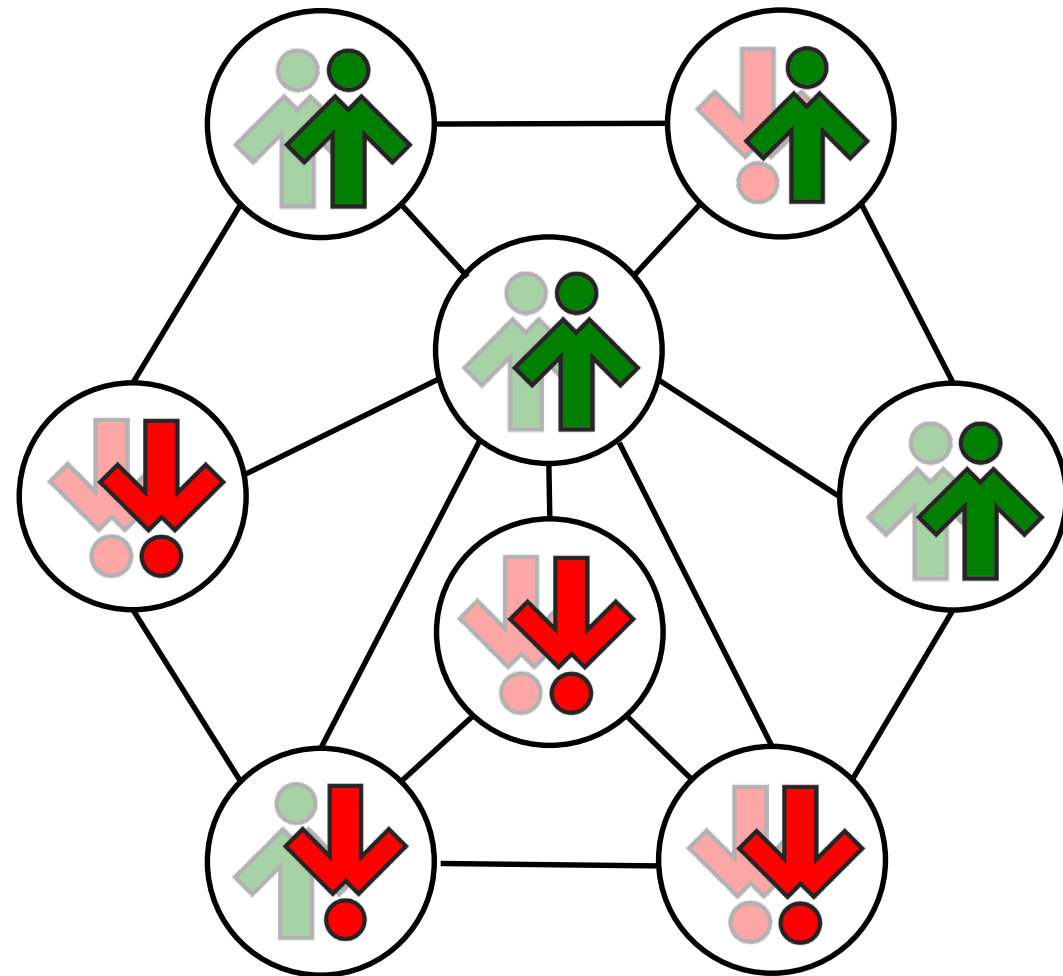
Arkadiusz Jędrzejewski¹, Grzegorz Marcjasz², Paul R. Nail³, Katarzyna Sznajd-Weron^{1*}

¹ Faculty of Fundamental Problems of Technology, Wrocław University of Science and Technology, Wrocław, Poland, ² Faculty of Pure and Applied Mathematics, Wrocław University of Science and Technology, Wrocław, Poland, ³ Faculty of Psychology and Counseling, University of Central Arkansas, Conway, Arkansas, United States of America



Act then **T**hink (AT) model

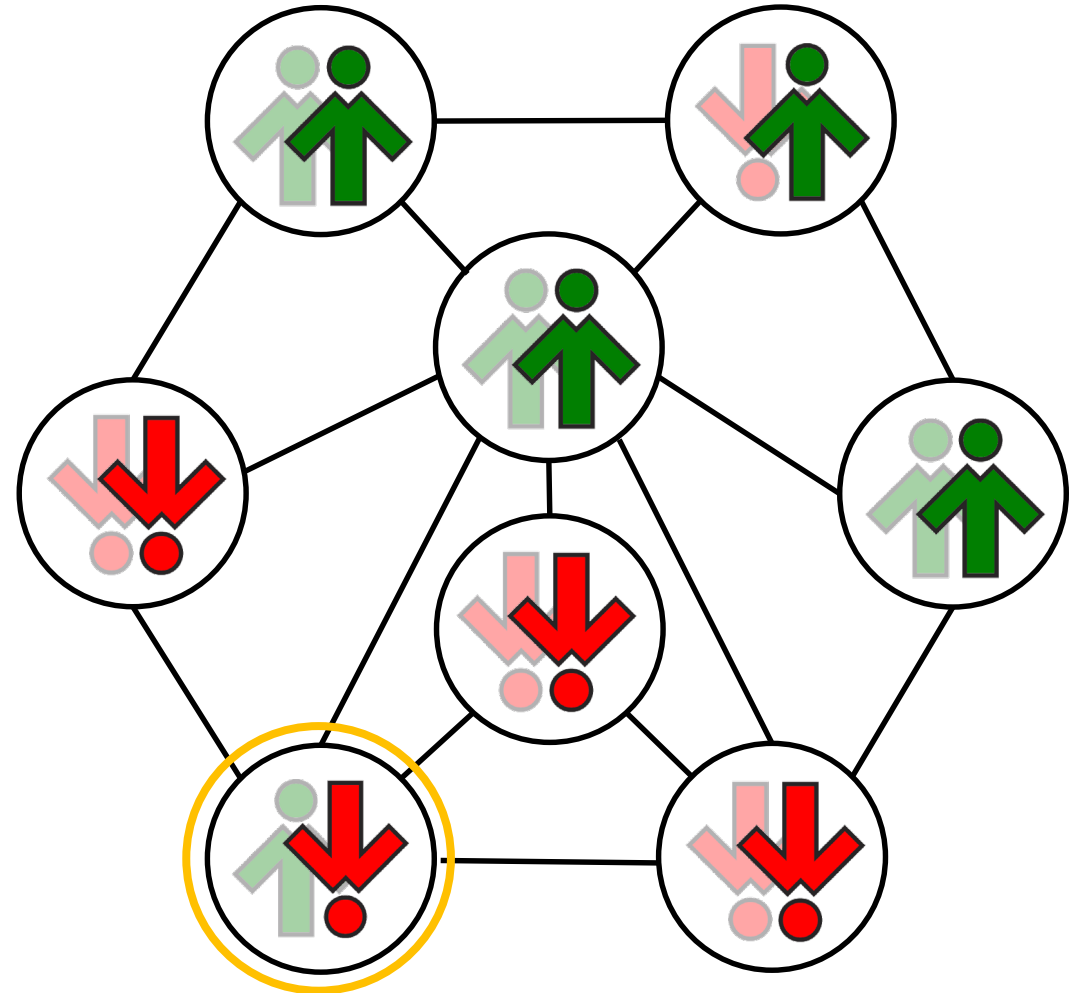
- choose one voter at random, located at site i



Act then **T**hink (AT) model

- choose one voter at random, located at site i
- Act: update the **public** opinion S_i
 - **Independence with prob p** : replace public opinion by the private one

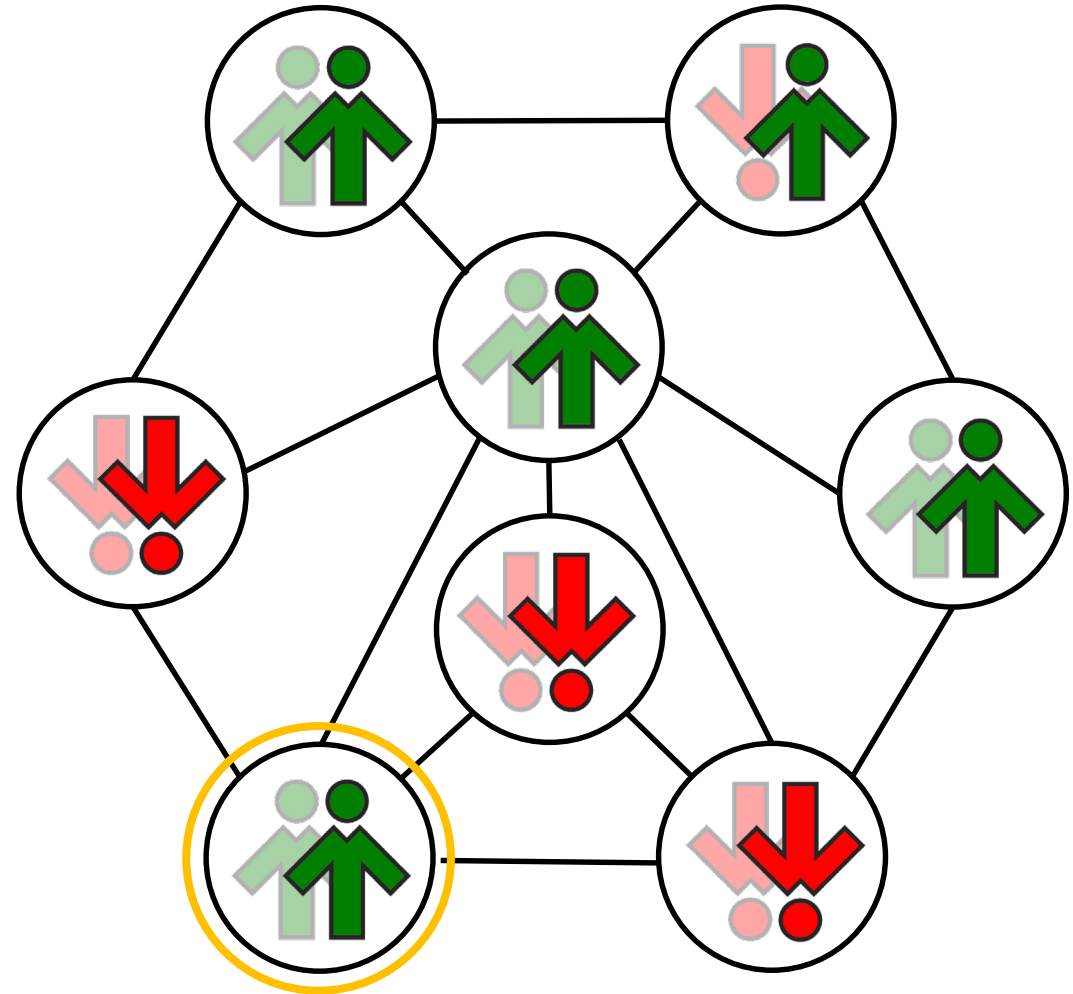
$$S_i \rightarrow \sigma_i$$



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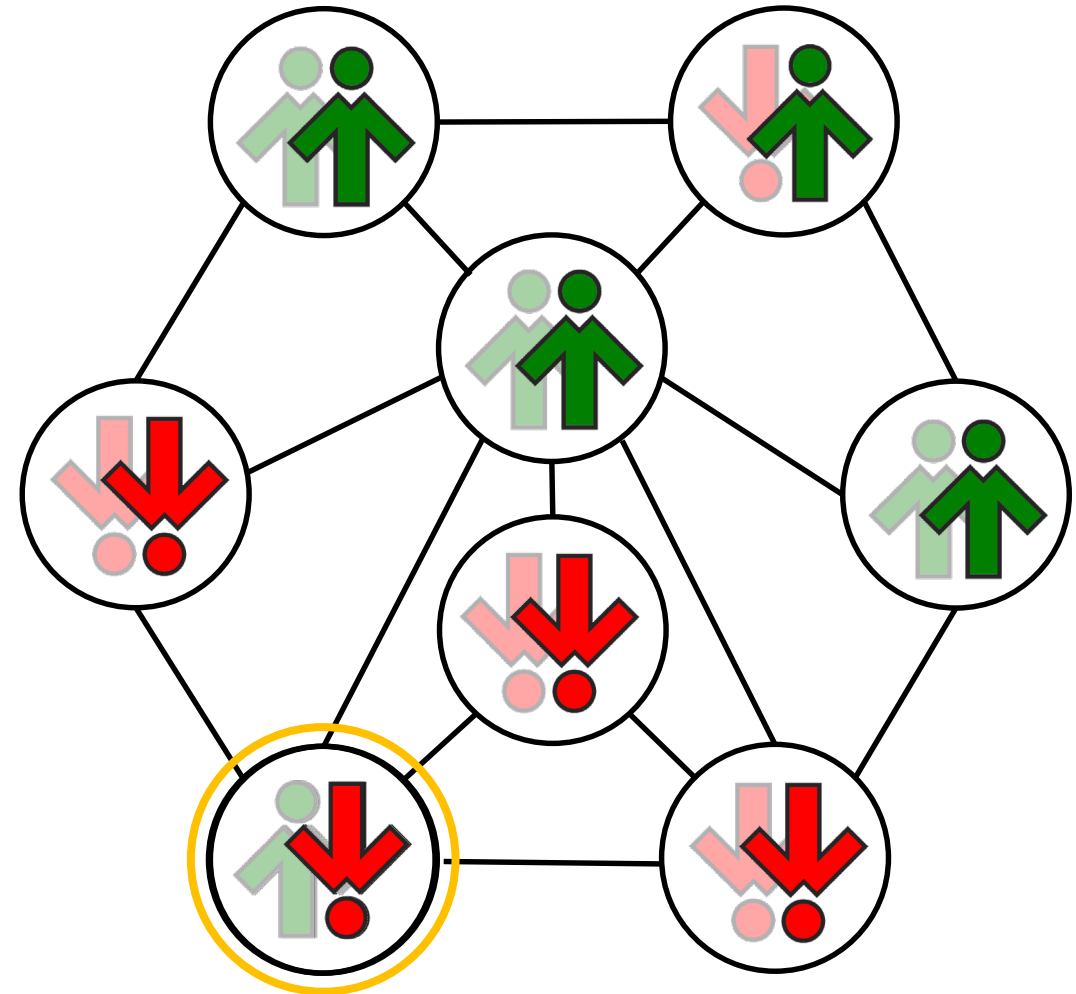


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 - 1) pick randomly q neighbours without repetition



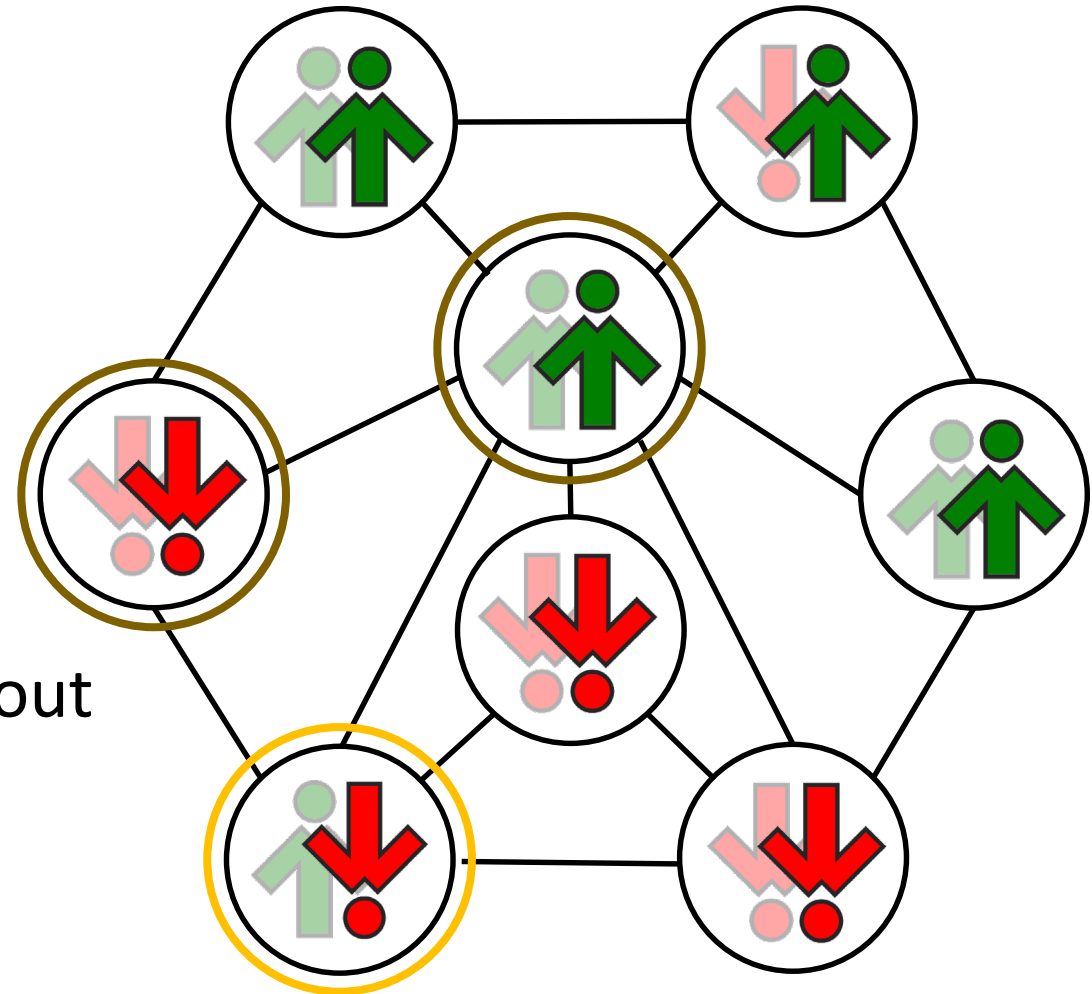
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 - 1) pick randomly q neighbours without repetition
 - 2) $S_i = \sigma_i$? NO: disinhibitory contagion $S_i \rightarrow \sigma_i$ if one $S_{ix} = \sigma_i$

Ex: $q = 2$



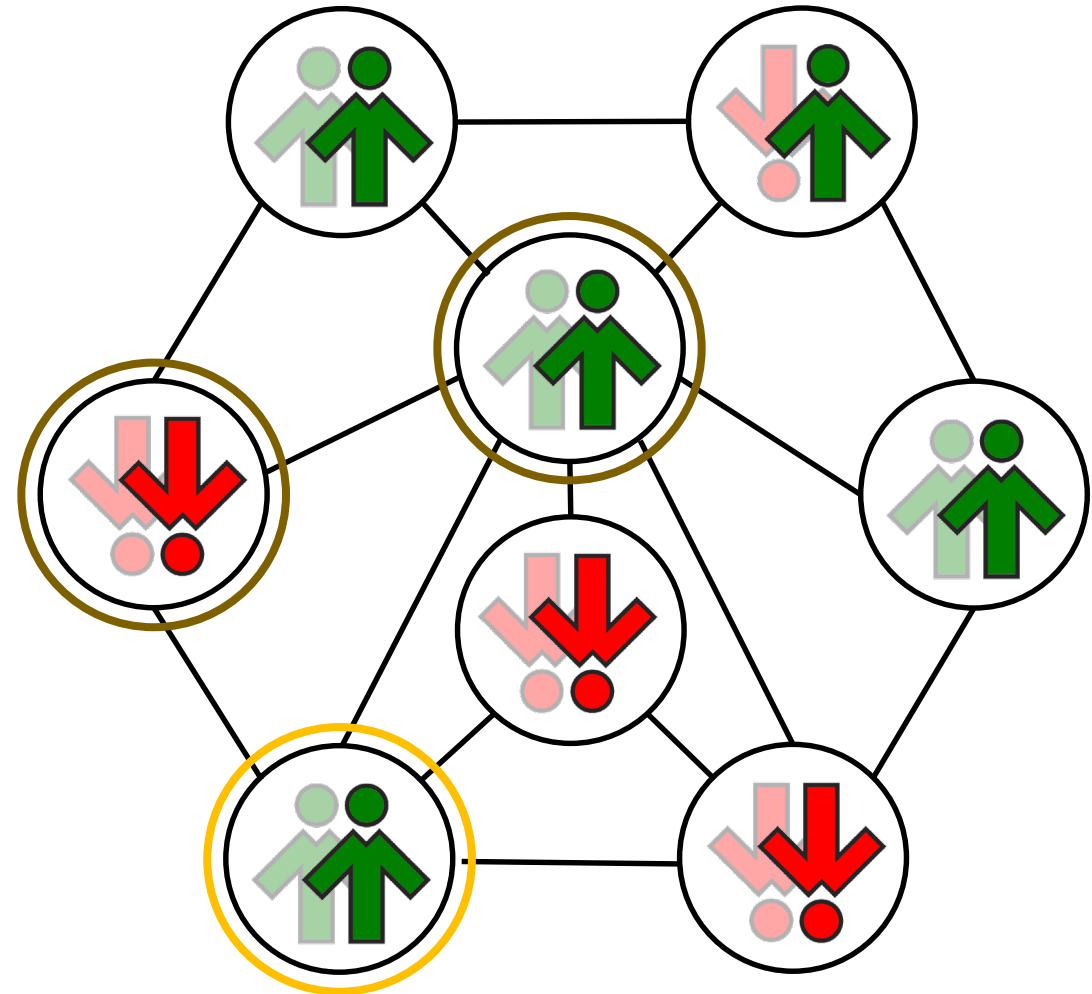
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 - 2) $S_i = \sigma_i$? NO: disinhibitory contagion $S_i \rightarrow \sigma_i$ if one $S_{ix} = \sigma_i$

Ex: $q = 2$



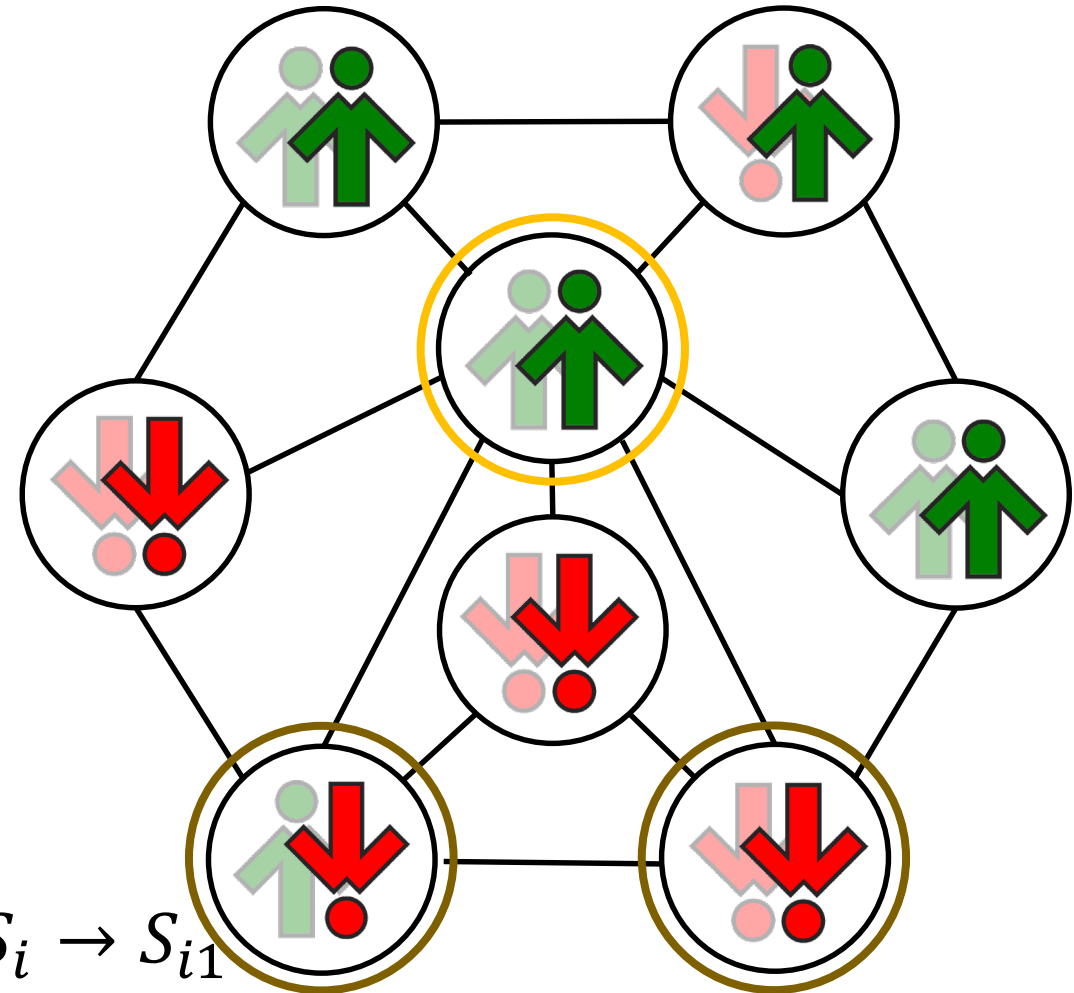
Act then **T**hink (AT) model

- choose one voter at random, located at site i
- Act: update the **public** opinion S_i
 - **Independence with prob p** : replace public opinion by the private one

$$S_i \rightarrow \sigma_i$$

- **Conformity with prob $1 - p$** :
 - 1) pick randomly q neighbours without repetition
 - 2) $S_i = \sigma_i$? YES
 - 3) unanimous: $S_{i_1} = \dots = S_{i_q}$? YES: $S_i \rightarrow S_{i_1}$

Ex: $q = 2$



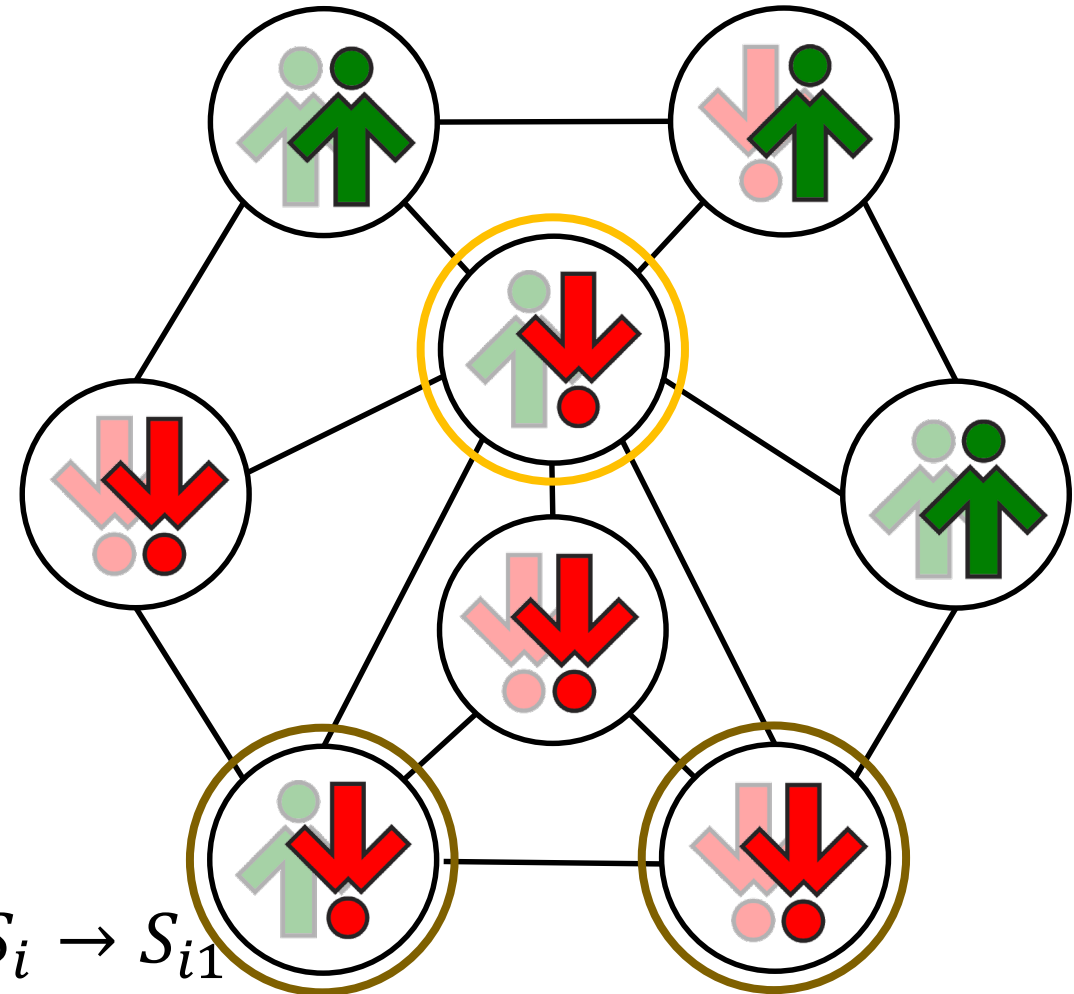
Act then **T**hink (AT) model

- choose one voter at random, located at site i
- Act: update the **public** opinion S_i
 - **Independence with prob p** : replace public opinion by the private one

$$S_i \rightarrow \sigma_i$$

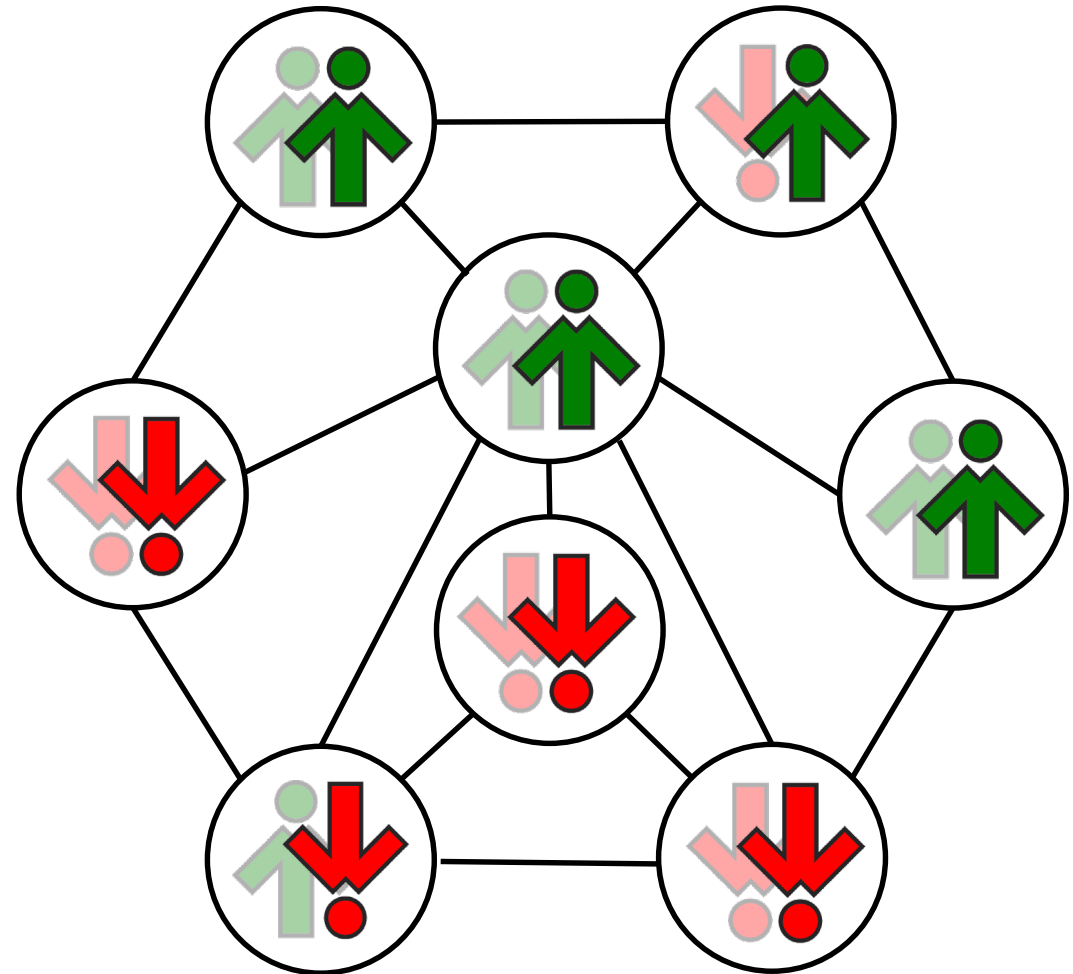
- **Conformity with prob $1 - p$** :
 - 1) pick randomly q neighbours without repetition
 - 2) $S_i = \sigma_i$? YES
 - 3) unanimous: $S_{i_1} = \dots = S_{i_q}$? YES: $S_i \rightarrow S_{i_1}$

Ex: $q = 2$



Act then **T**hink (AT) model

- choose one voter at random, located at site i
- Act: update the **public** opinion S_i
- Think: update the **private** opinion σ_i



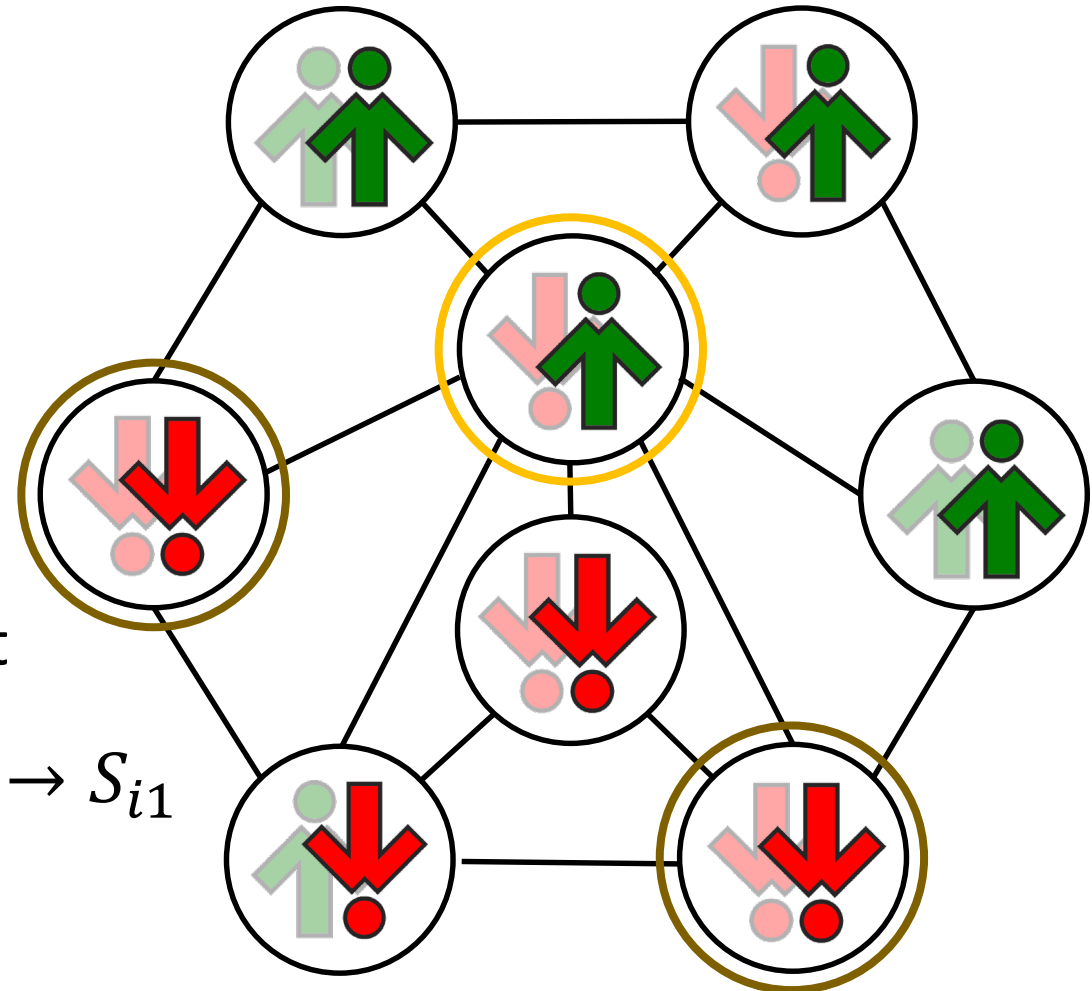
Act then Think (AT) model

- choose one voter at random, located at site i
- Act: update the **public** opinion S_i
- Think: update the **private** opinion σ_i
 - **Independence with prob p**

$$\sigma_i \xrightarrow{1/2} -\sigma_i$$

- **Conformity with prob $1 - p$:**
 - 1) pick randomly q neighbours without repetition
 - 2) unanimous: $S_{i_1} = \dots = S_{i_q}$? YES: $\sigma_i \rightarrow S_{i_1}$

Ex: $q = 2$





Why „Act then Think (AT) model”?

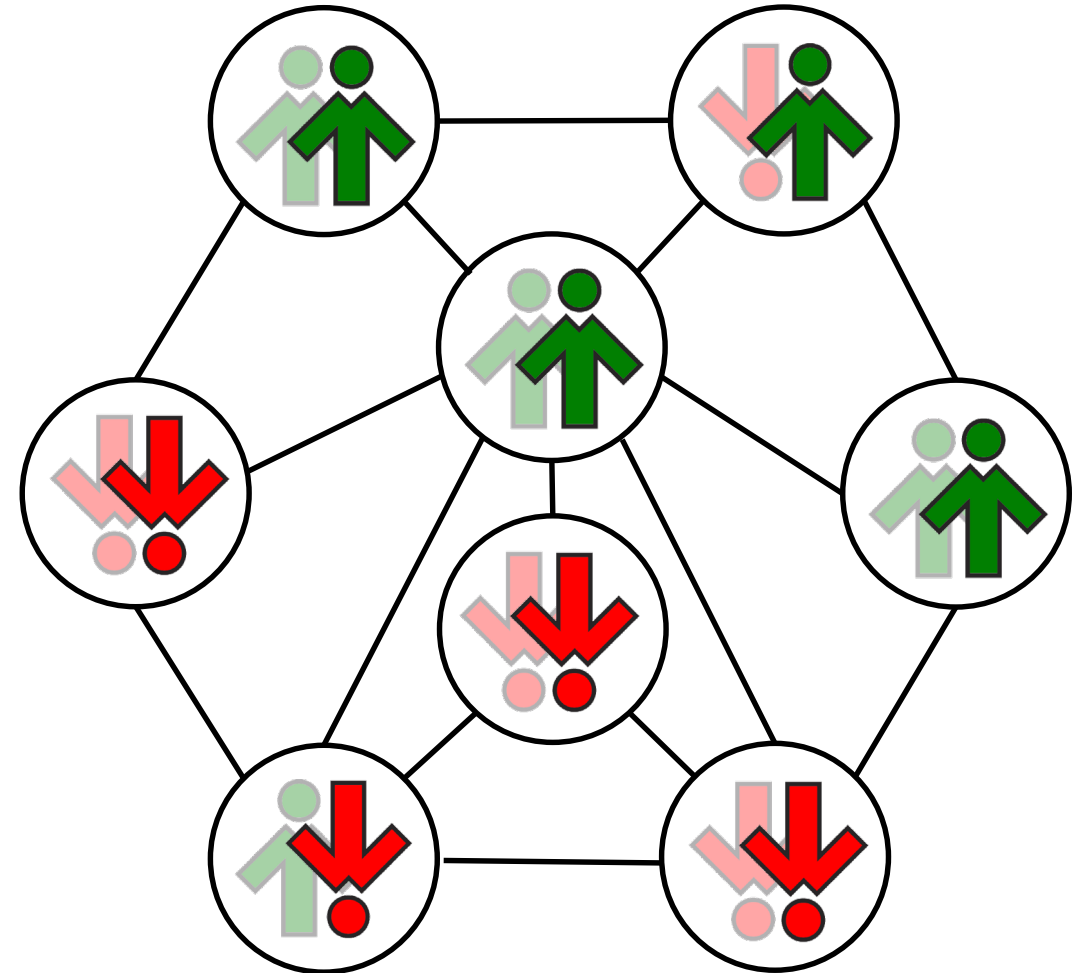
- *“If social psychology has taught us anything during the last 25 years, it is that we are likely not only to think ourselves into a way of acting but also to act ourselves into a way of thinking.”*

[David G. Myers, Social Psychology 10th Ed. page 131]

- How to update opinions?
- What should be updated first?
- Does it matter for ABM?

Two versions of the model: AT vs. TA

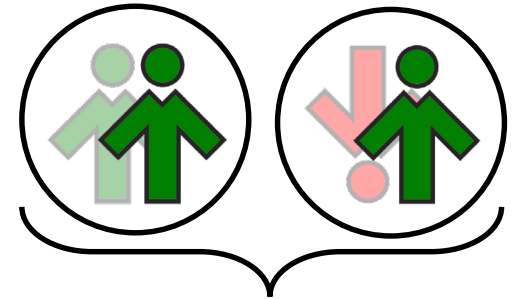
- **A**ct then **T**hink (AT) model
 - choose one voter at random, located at site i
 - Act: update the **public** opinion S_i
 - Think: update the **private** opinion σ_i
- **T**hink then **A**ct (TA) model
 - choose one voter at random, located at site i
 - Think: update the **private** opinion σ_i
 - Act: update the **public** opinion S_i



What do we measure?

- The fraction of individuals with the positive public opinion:

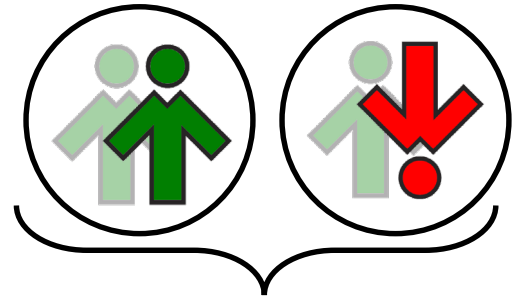
$$c_S(t) = \frac{N_{S=1}(t)}{N}$$



$$S_i(t) = 1$$

- The fraction of individuals with the positive private opinion:

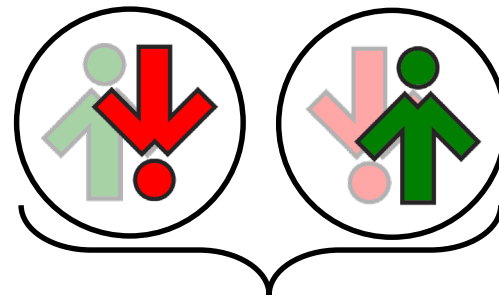
$$c_\sigma(t) = \frac{N_{\sigma=1}(t)}{N}$$



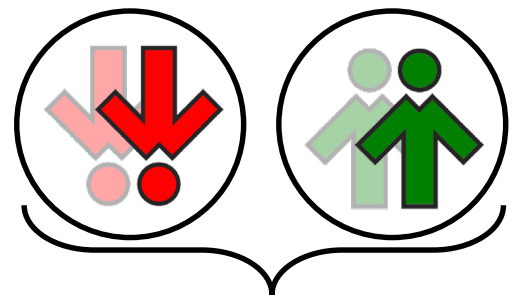
$$\sigma_i(t) = 1$$

- The level of dissonance = the fraction of individuals that have different public and private opinions:

$$d(t) = \frac{1}{2N} \sum_{i=1}^N (1 - S_i(t)\sigma_i(t))$$



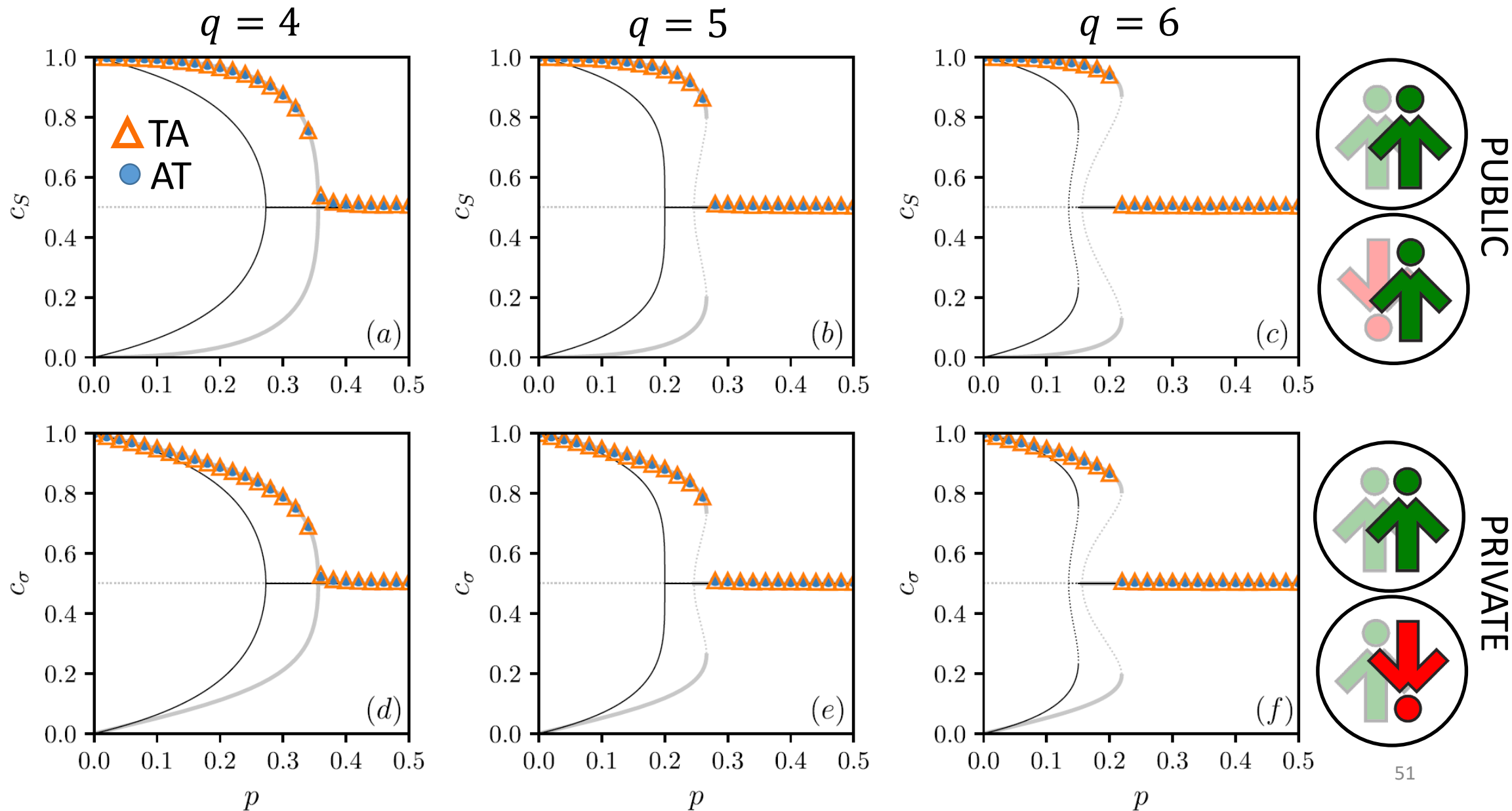
$$S_i(t)\sigma_i(t) = -1$$



$$S_i(t)\sigma_i(t) = 1$$



Stationary concentrations of positive opinions



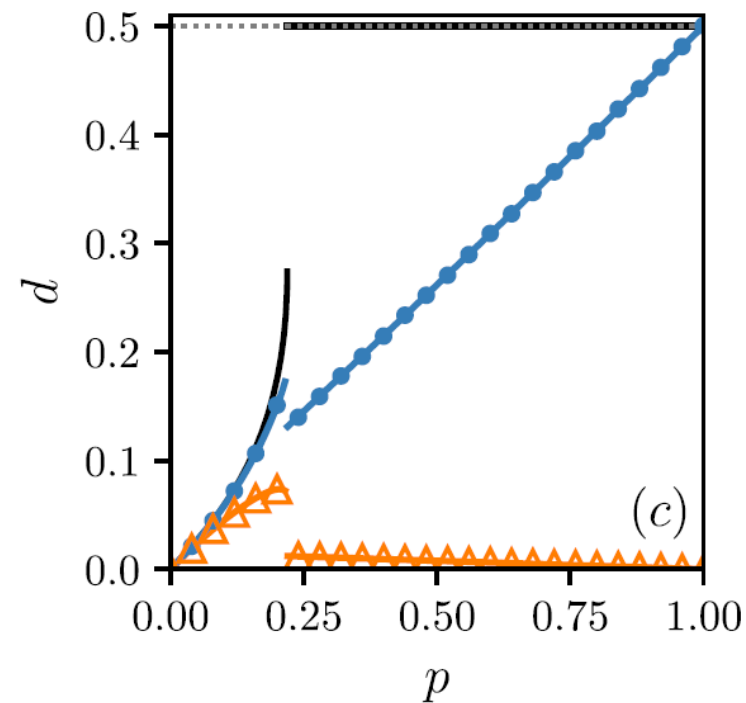
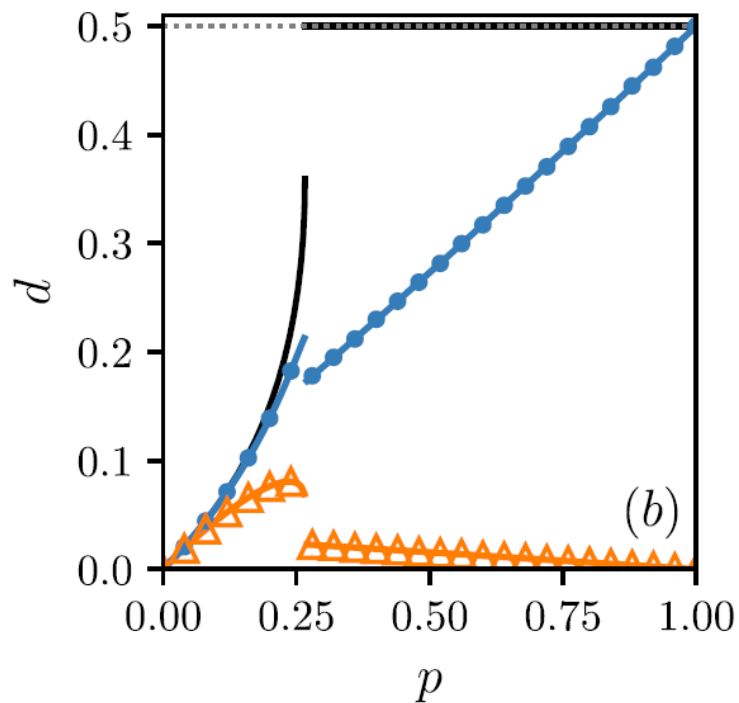
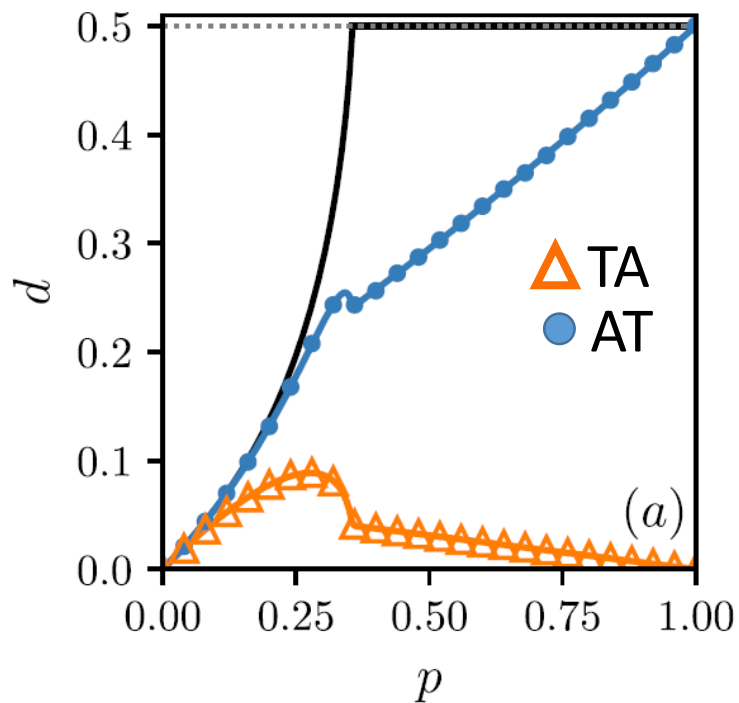


Stationary value of the dissonance

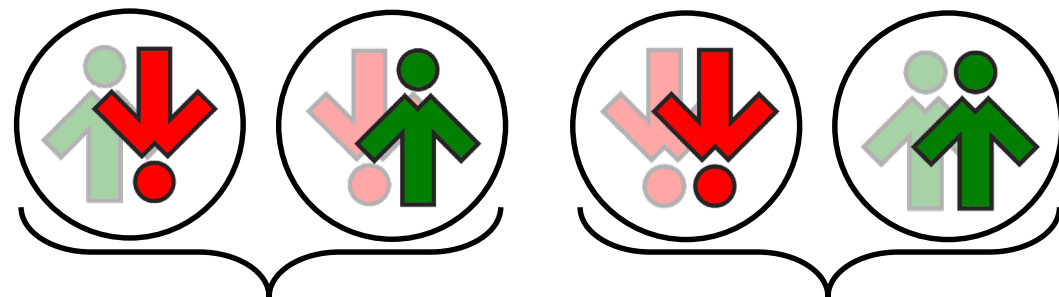
$q = 4$

$q = 5$

$q = 6$

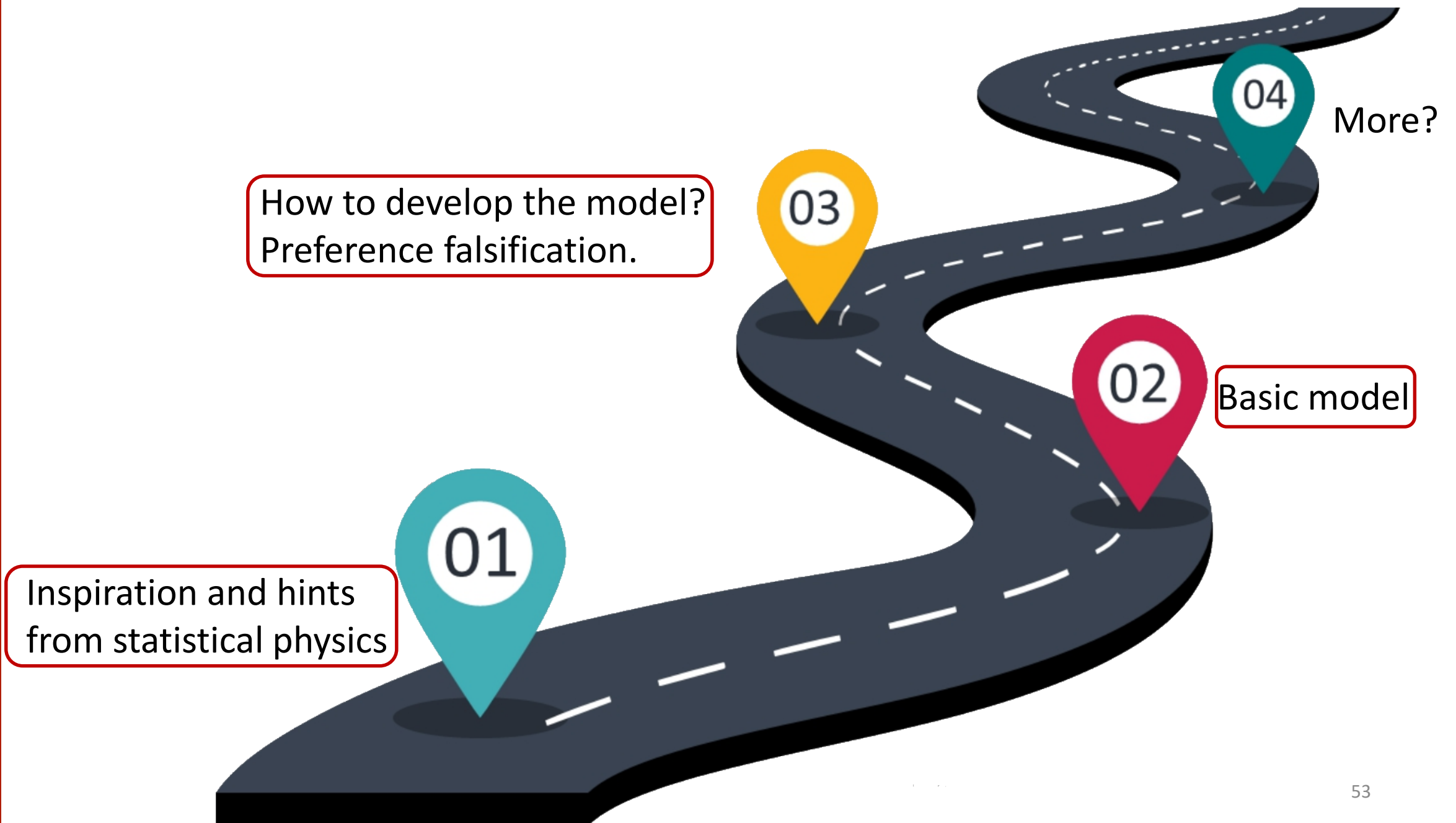


$$d(t) = \frac{1}{2N} \sum_{i=1}^N (1 - S_i(t)\sigma_i(t))$$



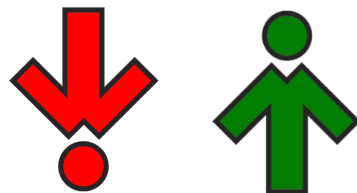
$$S_i(t)\sigma_i(t) = -1$$

$$S_i(t)\sigma_i(t) = 1$$



What factors promote social hysteresis in ABMs? (not only in the q-voter model - universal)

- Larger size of the group of influence
- Super majority (simple majority is not enough)
- Higher level of independence in the society
- More layers in the social network (Face-to-face, Facebook)
- Situation over personality (person-situation debate)
- Larger number of individual states on the public level
- Preference falsification (internal and external opinion)
- ...





Take-home messages

1. Memory can appear even in the society of memoryless agents.
2. More channels of communications, more independence and larger groups of influence promote social hysteresis.



Lecture based on:

1. B. Nowak, B. Stoń, KSW, Discontinuous phase transitions in the multi-state noisy q-voter model: quenched vs. annealed disorder, *Scientific Reports* (2021)
2. A Jędrzejewski, KSW, Statistical physics of opinion formation: Is it a SPOOF? *Comptes Rendus Physique* (2019)
3. B. Nowak and KSW, Homogeneous Symmetrical Threshold Model with Nonconformity: Independence versus Anticonformity, *Complexity* (2019)
4. P. Nyczka, K. Byrka, P. R. Nail and KSW, Conformity in numbers—Does criticality in social responses exist?, *PLoS ONE* (2018)
5. A. Jędrzejewski, G. Marcjasz, P. R. Nail, KSW, Think then act or act then think? *PLoS ONE* (2018)
6. A. Jędrzejewski, Pair approximation for the q-voter model with independence on complex networks, *PRE* (2017)
7. A Chmiel, KSW, Phase transitions in the q-voter model with noise on a duplex clique, *PRE* (2015)
8. P. Nyczka, KSW, *Anticonformity or Independence? -Insights from Statistical Physics*, *J. Stat. Phys.* (2013)
9. P. Nyczka, KSW, J. Cisko, *Phase transitions in the q-voter model with two types of stochastic driving*, *PRE* (2012)