Forecast averaging as a method to mitigate risks related to decision making in an energy company

Jakub Nowotarski

Supervisor: prof. dr hab. Rafał Weron Auxiliary supervisor: dr Katarzyna Maciejowska

Department of Operations Research Faculty of Computer Science and Management

Jakub Nowotarski

Forecast averaging as a method...

Wrocław, 01.06.2017 1 / 37

Jakub Nowotarski – a short bio

- 2008-2011 BSc in Mathematics, PWR
- 2011-2013 MSc in Financial & Actuarial Mathematics, PWR
 - Rector's Award for academic excellence
 - **Motivational Scholarship** from the Ministry of Science and Higher Education (MNiSW)
 - Participated in 11 modeling weeks (ECMI Modeling Week or European Study Group with Industry)
 - In 2012 asked if he could join a 'real research project'
 - In 2013 first article in Energy Economics

Jakub Nowotarski – a short bio

• 2013-2017 PhD in Management Science, PWR

- PRELUDIUM grant, National Science Centre, 07/2014
- Best Ph.D. student paper and presentation at the Conference on Energy Finance (EF14), Erice, Italy, 09/2014
- **2nd place** in the Global Energy Forecasting Competition, Probabilistic Price Forecasting, 07/2015
- Wincenty Styś scholarship for social and humanistic sciences, from the Mayor of Wrocław, 09/2015-06/2016
- Doctoral scholarship, MNiSW, 12/2016
 ⇒ the only one for an Economics student!

- 4 回 ト 4 ヨ ト 4 ヨ ト - -

Jakub Nowotarski – a short bio

• 2013-2017 PhD in Management Science, PWR, cont.

- Presented research on 17 conferences in Europe and USA
 - Conference on Energy Finance (2014, 2015, 2016)
 - IEEE PES (Power & Energy Society) General Meeting (2015)
 - International Symposium on Forecasting (2015, 2016)
- 16 publications
 - 12 on energy forecasting
 - 13 in JCR-listed journals
- 50 Scopus-indexed citations (w/o autocitations), H-index = 5
- Currently employed in the Model Risk Management Group at BNY Mellon, Wrocław

- ロ ト - (同 ト - (三 ト - (三 ト -)

Forecast averaging as a method to mitigate risks related to decision making in an energy company

Jakub Nowotarski

Supervisor: prof. dr hab. Rafał Weron Auxiliary supervisor: dr Katarzyna Maciejowska

Department of Operations Research Faculty of Computer Science and Management

Jakub Nowotarski

Forecast averaging as a method...

Wrocław, 01.06.2017 5 / 37

Electricity markets in Europe



... in North America and Australia



Forecast averaging as a method...

Day-ahead market and the 'spot' price



Construction of the spot price

Supply and demand, renewables and negative prices



Source: Ziel & Steinert (2016, Energy Economics)

9 / 37

< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

Electricity price time series

Seasonality, mean-reversion and price spikes



Importance of electricity price forecasting (EPF)

 Price and load forecasting errors reduced by 1% → savings of ca. \$600,000 per year (for 1 GW capacity):



New trends in EPF: forecast averaging

- Forecast averaging introduced in late 1960s (Bates & Granger, 1969; Crane & Crotty, 1967)
- In electricity markets
 - In load forecasting
 - since mid 1980s (Bunn, 1985; Bunn & Farmer, 1985; Smith, 1989), also applied recently (Loland et al., 2012; Taylor, 2010; Taylor & Majithia, 2000)
 - First EPF papers appeared only in 2013: Bordignon et al. (2013), Nowotarski et al. (2014), Weron (2014), Raviv et al. (2015)

New trends in EPF: probabilistic forecasting

- Prediction intervals or density forecasts include more comprehensive information
- Can be used by managers in planning and decision making (Chatfield, 2000; Gneiting & Katzfuss, 2014)



Aims and objectives

Aim: develop efficient and robust forecasting techniques for spot prices in day-ahead electricity markets

Objectives:

- Verification of whether a forecast combination may improve the quality [P1, P2-P3]
- Development of new approaches to probabilistic forecasting [P2, P3-P4]
- Presenting guidelines for the rigorous use of methods, measures and tests for probabilistic price forecast evaluation [P5]

[P1] Forecast averaging (point forecasts)



An empirical comparison of alternative schemes for combining electricity spot price forecasts



Jakub Nowotarski^a, Eran Raviv^b, Stefan Trück^c, Rafał Weron^{a,*}

a Institute of Organization and Management, Wrocław University of Technology, Wrocław, Poland

^b Department of Econometrics, Erasmus University, Rotterdam, The Netherlands

^c Faculty of Business and Economics, Macquarie University, Sydney, Australia

ARTICLE INFO

Article history: Received 20 August 2013 Received in revised form 14 May 2014 Accepted 19 July 2014 Available online 31 July 2014

JEL Codes: C22 Time-Series Models C52 Model Evaluation, Validation, and Selection C53 Forecasting and Prediction Methods L94 Electric Utilities Q47 Energy Forecasting

ABSTRACT

In this comprehensive empirical study we critically evaluate the use of forecast averaging in the context of electricity prices we apoly seem averaging and one selection scheme and perform a backetsing analysis on day-ahead electricity prices in three major European and US markets. Our findings support the additional benefit of combining fore-tasts of individual methods for deriving more accurate predictions, however, the performance is not uniform across the considered markets and periods. In particular, equally weighted pooling of forecasts emerges as a simple, yet powerful technique compared with other schemes that rely on estimated combination weights, but only when there is no individual predictor that consistently outperforms its competitors. Constrained least squares regression (CLS) offers a balance between robustness against such well performing individual methods and relatively accurate forecasts, on average better than those of the individual predictors. Finally, some popular forecast averaging schemes – like ordinary least squares regression (OLS) and Bayesian Model Averaging (BMA) – turn out to be unsuitable for predicting day-hadea electricity prices.

イロト イボト イヨト イヨト

[P1] What is forecast averaging?



< 17 ►

3

[P1] Results

- Empirical 'evidence' that forecast averaging works for electricity spot prices
- Improvements statistically significant (Diebold-Mariano test)

| Week | Simple | OLS | LAD | PW | CLS | IRMSE | BMA | BI | ARX |
|--------------------|--------|------|------|------|------|-------|------|------|------|
| Summary statistics | | | | | | | | | |
| WMAE | 8.25 | 9.58 | 7.80 | 8.58 | 8.57 | 8.26 | 9.54 | 8.50 | 9.42 |
| # better than AR | 27 | 17 | 26 | 19 | 27 | 27 | 17 | 22 | - |
| # better than BI | 15 | 8 | 21 | 14 | 14 | 15 | 8 | - | - |
| # best | 1 | 2 | 13 | 5 | 1 | 4 | 0 | 4 | - |
| m.d.f.b. | 1.07 | 2.40 | 0.62 | 1.39 | 1.39 | 1.07 | 2.36 | 1.31 | 2.24 |

Nord Pool 2010

17 / 37

< 4 P→

[P2-4] Forecast averaging (probabilistic forecasts)

Comput Stat (2015) 30:791-803 DOI 10.1007/s00180-014-0523-0



ORIGINAL PAPER



[P2] Computing electricity spot price prediction intervals using quantile regression and forecast averaging

Jakub Nowotarski · Rafał Weron

Abstract We examine possible accuracy gains from forecast averaging in the context of interval forecasts of electricity spot prices. First, we test whether constructing empirical prediction intervals (PI) from combined electricity spot price forecasts leads



Hybrid model

Noball function

Quantile remession

Electricity spot price

Prediction interval Quantile regression Factor model Electricity spot price

forecast averaging to compute interval forecasts of electricity spot prices. We extend the Ouantile Regression Averaging (ORA) approach of Nowotarski and Weron (2014a), and use principal component analysis to automate the process of selecting from among a large set of individual forecasting models that are available for averaging. We show that the resulting Factor Quantile Regression Averaging (FORA) approach performs very well for price (and load) data from the British power market. In terms of unconditional coverage, conditional coverage and the Winkler score, we find the FQRA-implied prediction intervals to be more accurate than those of either the benchmark ARX model or the QRA approach.

© 2014 International Institute of Forecasters, Published by Elsevier B.V. All rights reserved.

electricity price forecasting track of GEFCom2014. A new hybrid model extending the Quantile Regression Averaging (QRA) approach of Nowotarski and Weron (2015) is proposed. It consists of four major blocks: point forecasting, pre-filtering, quantile regression modeling and post-processing. This universal model structure enables a single block to be developed independently, without the performances of the remaining blocks being affected. The four-block model design is complemented by the inclusion of expert judgement, which may be of great importance in periods of unusually high or low electricity demand. © 2015 International Institute of Forecasters, Published by Elsevier B.V. All rights reserved.

イロト イポト イヨト イヨト

Jakub Nowotarski

Forecast averaging as a method...

Wrocław, 01.06.2017 18 / 37

[P2] Quantile Regression Averaging (QRA)

Point forecasts used for constructing probabilistic forecasts



19 / 37

[P3] Factor QRA (FQRA)

QRA for a large number of individual forecasts



Forecast averaging as a method...

Wrocław, 01.06.2017 20 / 37

[P4] QRA in GEFCom2014

1st and 2nd place for QRA!



Jakub Nowotarski

Forecast averaging as a method...

Wrocław, 01.06.2017

21 / 37

イロト 不得 トイラト イラト 二日

[P5] Guidelines for the use of methods, measures and tests



Recent advances in electricity price forecasting: A review of probabilistic forecasting

Jakub Nowotarskia, Rafał Werona

"Department of Operations Research, Wrocław University of Science and Technology, 50-370 Wrocław, Poland

Abstract

Since the inception of competitive power markets two decades ago, *electricity price forecas*ting (EPF) has gradually become a fundamental process for energy companies' decision making mechanisms. Over the years, the bulk of research has concerned point predictions. However, the recent introduction of smart grids and renewable integration requirements has had the effect of increasing the uncertainty of future supply, demand and prices. Academics and practitioners alke have come to understand that probabilistic electricity price (and load) forecasting is now more important for energy systems planning and operations than ever before. With this paper we offer a tutorial review of probabilistic EPF and present much needed guidelines for the rigorous use of methods, measures and tests, in line with the paradigm of 'maximizing sharpness subject to reliability'. The paper can be treated as an update and a further extension of the otherwise comprehensive EPF review of Weron [1] or as a standalone treatment of a fascinating and underdeveloped topic, that has a much broader reach than EPF itself.

Forecast averaging as a method...

22 / 37

[P5] Probabilistic forecast evaluation

• Nontrivial - we forecast a distribution but observe only one value



23 / 37

[P5] Probabilistic forecast evaluation

- A critical review of methods with recommendations
- Empirical analysis with practical examples

| Interval forecasts Statistics | Tests | Density forecasts Statistics | Tests | | | |
|---|--|--|--|--|--|--|
| Reliability / calibration / unbiasedness | | | | | | |
| Unconditional coverage [46, 74] | Kupiec 74 | Probability Integral Transform (PIT) 14.75 | Visual 'tests' 14.16 Tests for uniformity 176.77 | | | |
| Conditional coverage [46] (CC = UC + Independence) | Christoffersen 46 (Lagged 78) Ljung-Box Christoffersen 79 Duration-based tests 80 81 Dynamic Quantile (DQ) 82 VQR 83 | Berkowitz CC statistic 48 | Berkowitz 48 | | | |
| Sharpness (and reliability) | | | | | | |
| Pinball loss 84,85 Winkler (interval) score 86 | Diebold-Mariano 87,88 Model confidence set 89 Forecast encompassing 90 | Continuous Ranked Probabi- lity Score (CRPS) 15 91 Logarithmic score 192 | Diebold-Mariano 87,88 Model confidence set 89 Forecast encompassing 90 | | | |

Key results

- Forecast averaging is more accurate and less uncertain than the best ex-ante selected method [P1, P2-P3]
- Quantile Regression Averaging (QRA) a new, highly accurate method for probabilistic forecasting which merges forecast averaging and quantile regression [P2, P3-P4]
- Original set of guidelines for use of probabilistic forecast evaluation [P5]

25 / 37

Publications

- J. Nowotarski, E. Raviv, S. Trück, R. Weron (2014) An empirical comparison of alternate schemes for combining electricity spot price forecasts, Energy Economics 46, 395-412, [IF_{5Y}=3.574, 40p MNiSW]
- J. Nowotarski, R. Weron (2015) Computing electricity spot price prediction intervals using quantile regression and forecast averaging, Computational Statistics 30(3), 791-803, [IF_{5Y}=0.560, 15p MNiSW]
- K. Maciejowska, J. Nowotarski, R. Weron (2014) Probabilitic forecasting of electricity spot prices, International Journal of Forecasting 32, 957-965, [IF_{5Y}=1.994, 30p MNiSW]
- K. Maciejowska, J. Nowotarski (2016), A hybrid model for GEFCom2014 probabilistic electricity price forecasting, International Journal of Forecasting 32 (3), 1051-1056, [IF_{5Y}=1.994, 30p MNiSW]
- J. Nowotarski, R. Weron (2017), Recent advances in electricity price forecasting: A review of probabilistic forecasting, Renewable & Sustainable Energy Reviews, forthcoming, [IF_{5Y}=7.896, 45p MNiSW]
- 9 more publications on energy forecasting (including 5 JCR-listed)
- 3 JCR-listed publications on statistical methods in medicine

26 / 37

イロト 不得 トイラト イラト 二日

Replies to reviews

Jakub Nowotarski

Forecast averaging as a method...

✓ □→ < ≥→ < ≥→ <
 Wrocław, 01.06.2017

э

27 / 37

1. Contributions to decision making

- Electricity price forecasts are used as an input to decision making models (Conejo et al., 2010; Shahidehpour et al., 2002):
 - DA vs RT/balancing market bidding (arbitrage) strategy
 - Bidding strategy of a wind producer (additional wind uncertainty)
 - Risk management for a wind producer with a penalty applied to a risk measure
- More accurate model inputs lead to better decisions

References:

Conejo et al. (2010) Decision Making Under Uncertainty in Electricity Markets, Springer Shahidehpour et al. (2002) Market operations in electric power systems, Wiley

Jakub Nowotarski

Wrocław, 01.06.2017 28 / 37

2. Financial impact of accurate forecasting

- Hong (2015): back-of-the-envelope calculation (assumes knowing the relation between DA and RT prices)
 ⇒ \$600,000 annual savings
- Zareipour et al. (2010): relative cost difference between optimizations with real and forecasted prices
 - Up to 0.35% cost reductions per 1% of MAPE
 - Based on Forecast Inaccuracy Economic Impact:

 $\frac{\text{Cost(Forecasted Price)} - \text{Cost(Actual Price)}}{\text{Cost(Forecasted Price)}} \cdot 100\%$

References:

Hong (2015) Crystal ball lessons in predictive analytics, EnergyBiz Magazine Zareipour et al. (2010) Economic impact of electricity market price forecasting errors: A demand-side analysis, IEEE Transactions on Power Systems

Jakub Nowotarski

Wrocław, 01.06.2017 29 / 37

2. Financial impact of accurate forecasting cont.

• Boomsmaa et al. (2014): 14% gain with forecasting and a specific optimization scheme

| | Spot price (Euro/ megawatt hour) | Bal. price (Euro/ megawatt hour) | Up-reg. price (Euro) | Down-reg. price (Euro) | Profit, spot (Euro) | Profit, sep. (Euro) | Profit, cor. (Euro) | Gain, pct. (%) |
|-----------|-------------------------------------|-------------------------------------|-------------------------|---------------------------|---------------------------|---------------------------|------------------------|-------------------|
| January | 50.83 (18.89) | 42.80 (23.60) | 18.44 | 24.36 | 7888.43 | 8083.54 | 9715.40 | 20.19 |
| February | 37.81 (17.48) | 32.81 (23.20) | 18.87 | 13.93 | 6804.23 | 8149.27 | 8149.27 | 14.71 |
| March | 30.53 (18.09) | 23.28 (22.61) | 13.22 | 10.06 | 6275.98 | 6580.84 | 8208.51 | 24.73 |
| April | 49.81 (16.98) | 42.12 (22.47) | 19.00 | 23.11 | 7784.69 | 8054.00 | 9517.38 | 18.17 |
| May | 40.02 (17.18) | 35.30 (21.71) | 19.45 | 15.85 | 6969.11 | 7227.26 | 8269.78 | 14.42 |
| June | 52.71 (18.39) | 48.97 (22.65) | 25.50 | 23.46 | 8064.44 | 8205.15 | 9016.59 | 9.89 |
| July | 45.09 (16.25) | 41.24 (22.36) | 23.20 | 18.04 | 7378.47 | 7593.78 | 8374.49 | 10.28 |
| August | 40.20 (16.22) | 35.79 (22.10) | 20.45 | 15.34 | 6968.93 | 7323.68 | 8175.58 | 11.63 |
| September | 47.65 (15.10) | 42.76 (20.43) | 21.88 | 20.89 | 7587.45 | 7711.59 | 8782.73 | 13.89 |
| October | 54.37 (17.43) | 50.15 (22.17) | 25.37 | 24.78 | 8190.54 | 8322.00 | 9247.50 | 11.12 |
| November | 58.64 (16.62) | 54.11 (22.37) | 27.55 | 26.56 | 8578.88 | 8681.39 | 9647.77 | 11.13 |
| December | 72.27 (21.22) | 68.05 (25.83) | 32.40 | 35.65 | 9831.01 | 9974.00 | 10774.91 | 8.03 |

Reference:

Boomsmaa et al. (2014) Bidding in sequential electricity markets: The Nordic case, European Journal of Operational Research

30 / 37

イロト イポト イヨト イヨト

3. When are the forecasts actually made?

- Most of the datasets assume forecasting 12-36 hours ahead
- GEFCom2014's source has not been revealed
- Exogenous variables: real temperatures, forecasted temperatures, forecasted loads
- Reason: compromise between complexity and data availability



4. Simplicity of individual models

- Linear relationship may not capture all dependence
- Full diagnostics is nontrivial (\rightarrow other explanatory variables)
- Neural networks do not outperform ARX (Marcjasz et al., 2017)

$$p_{d,h} = \underbrace{\beta_{h,1}p_{d-1,h} + \beta_{h,2}p_{d-2,h} + \beta_{h,3}p_{d-7,h}}_{\text{autoregressive effects}} + \underbrace{\beta_{h,4}p_{d-1,\min}}_{\text{non-linear effect}} + \beta_{h,5}z_{t} + \underbrace{\sum_{i=1}^{3} \beta_{h,i+5}D_{i}}_{\text{weekday dummies}} + \varepsilon_{d,i}$$

$$\underbrace{\frac{\text{GEFCom2014}}{Benchmarks}}_{\text{Naïve} ARX ANN_{5}} + \underbrace{\frac{\text{Nord Pool}}{Benchmarks}}_{\text{Naïve} ARX ANN_{5}} + \underbrace{\frac{\text{Nord Pool}}{Benchmarks}}_{\text{Nord Pool}} + \underbrace{\frac{\text{Nord Pool}}{Benchmarks}}$$

Reference:

Marcjasz et al. (2017) Importance of the long-term seasonal component in day-ahead electricity price forecasting revisited: Statistical vs. neural network models, Working Paper

ヘロト 不得下 イヨト イヨト

1. Title: Is mentioning risk management relevant?

- Suggested titles also reflect the content well, but ...
- Timmermann (2006): "[in forecasting] <u>uncertainty</u> is reflected in the forecast error and the source of <u>risk</u> reflects incomplete information about the target variable"
- Bunn (1985): Using forecast averaging may mitigate this risk

References:

Timmermann (2006) Forecast combinations, Handbook of Economic Forecasting Bunn (1985) Statistical efficiency in the linear combination of forecasts, Int. J. Forecasting

2. Properties of the underlying time series ...

- Hyndman & Athanasopoulos (2014): predictability of an event or a quantity depends on:
 - How well we understand the factors that contribute to it (bidding strategies, demand for electricity, weather)
 - How much data are available
 - Whether the forecasts can affect the thing we are trying to forecast
- Load forecasts satisfy all three conditions ...
- ... but price forecasts not really

Reference:

Hyndman & Athanasopoulos (2014) Forecasting: principles and practice, OTexts

イロト イポト イヨト イヨト

2. ... and combined forecast accuracy

- Empirical evidence shown in diverse areas
- The advantages of forecast combinations were listed by Timmermann (2006):
 - The full information set underlying the individual forecasts is unobserved
 - Individual forecasts may be differently affected by non-stationarities

Reference:

Timmermann (2006) Forecast combinations, Handbook of Economic Forecasting

3. The best averaging scheme for all datasets

- One of the goals in [P1]
- Backtesting exercise recommended to identify the preferred forecast averaging method
- Some studies consider even combining combinations
- ... but simple average is robust and hard to beat
 - Genre et al. (2013, IJF) *Combining expert forecasts: Can anything beat the simple average?*
 - DeMiguel et al. (2007, RFS) *Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?*

4. Theoretical properties of QRA

- QRA falls into a general class of asymmetric loss functions
- Elliott & Timmermann (2004) provide theoretical results
- Under some assumptions QRA is equivalent a 2-step procedure:
 - Estimate weights using OLS
 - ② Use the residuals from step 1 to estimate the constant in QRA ⇒ Accuracy of QRA related to the accuracy of point forecast combination
- Theoretical properties of quantile regression hold as well (Koenker, 2005)

References:

Elliott & Timmermann (2004) Optimal forecast combinations under general loss functions and forecast error distributions, Journal of Econometrics Koenker (2005) Quantile regression, Cambridge University Press

37 / 37

イロト イポト イヨト イヨト 二日