Modeling and forecasting electricity forward prices: A DSFM approach

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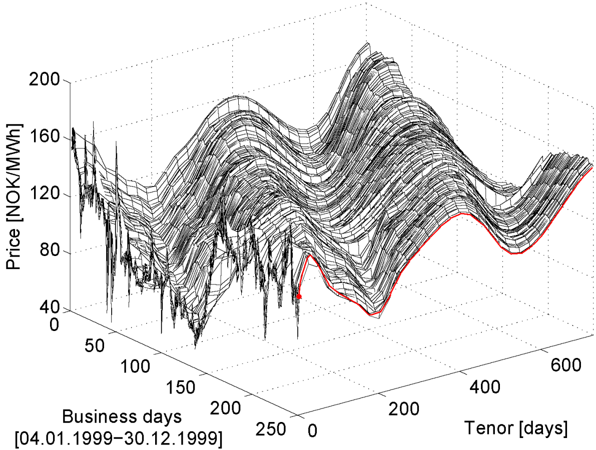
How can we model the dynamics of the electricity forward curve?



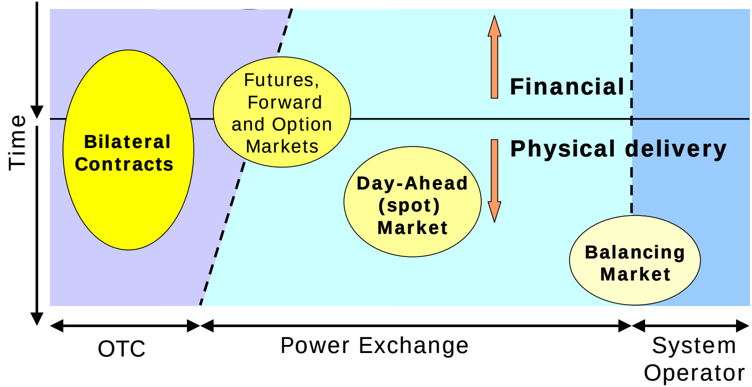
Motivation

- The electricity forward curve is a complex object with a non-trivial structure which exhibits **seasonality** and **extreme volatility at the short end**
- Our aim is to model and estimate forward curves for **trading**, **hedging** and **risk management**
- In this context the electricity forward curve acts as a very **high-dimensional** state variable
- Practice requires a **low-dimensional** representation of the curve

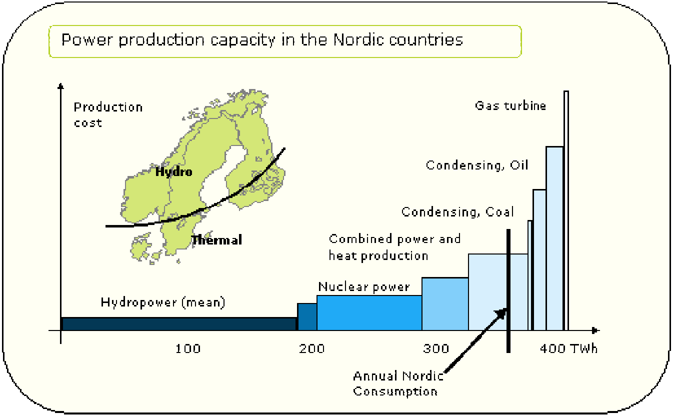
Agenda

1. Motivation ✓
2. **The Nordic power market and the forward curves**
3. The Dynamic Semiparametric Factor Model
4. Modeling and forecasting
5. Conclusions

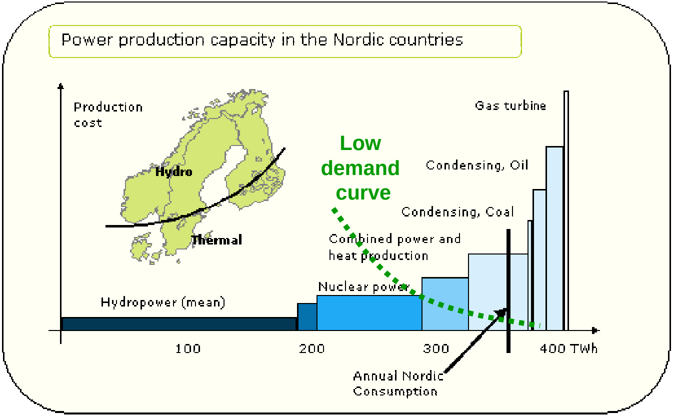
Wholesale power market structure



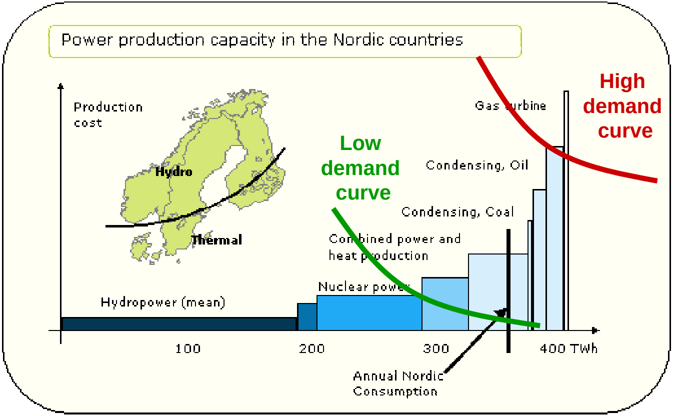
Supply stack and the market cross



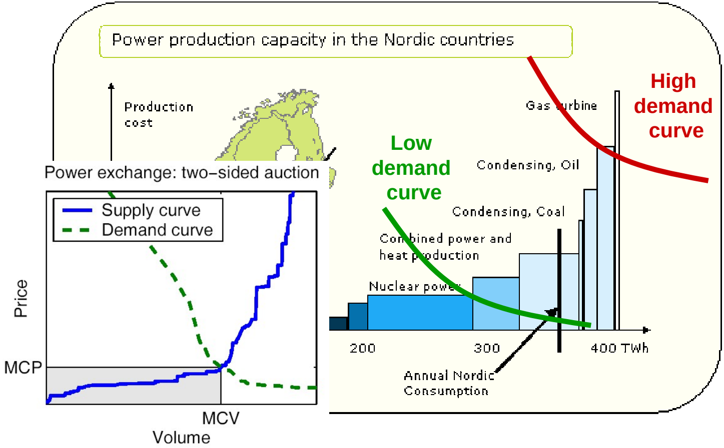
Supply stack and the market cross cont.



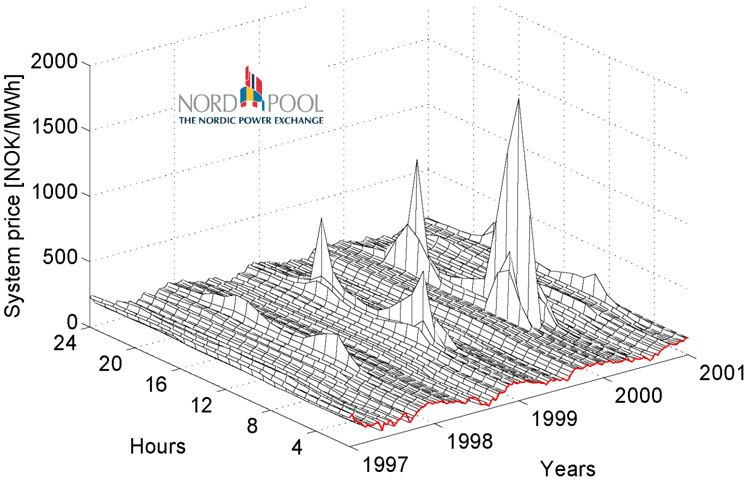
Supply stack and the market cross cont.



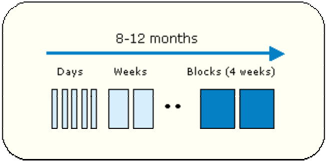
Supply stack and the market cross cont.



Seasonality, extreme volatility and spikes

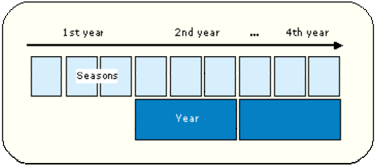


Futures and forwards at Nord Pool



Futures contracts

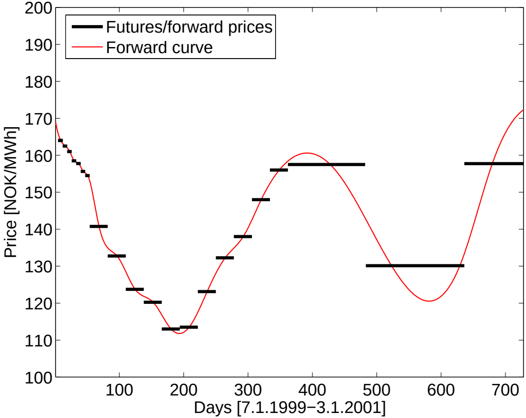
Day contracts (Dxx): 24 hours
Week contracts (GUxx): 7 days
Block contracts (GBxx): 4 weeks



Forward contracts

Winter 1 (FWV1xx): January - April
Summer (FWSOxx): May - September
Winter 2 (FWV2xx): October - December
Year (FWYRxx)

The forward curve ...



... and its dynamics

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Background

- The **Dynamic Semiparametric Factor Model (DSFM)** is a principal component type approach
- Primary application: Dimension reduction
- Originally introduced for modeling implied volatility surfaces [Fengler et al. (2007)]
- Can be seen as a combination of functional principal component analysis (fPCA) and nonparametric curve estimation

The model

- The **Dynamic Semiparametric Factor Model** has the form

$$\mathbf{Y}_t = m_0(\mathbf{X}_t) + \sum_{l=1}^{L} Z_{t,l} m_l(\mathbf{X}_t) + \varepsilon_t$$

  where the data vector $\mathbf{X}_t$ has J coordinates $X_{t,j}$ (observations per day)

- $\mathbf{m}(\cdot)$ is a tuple of basis functions $(m_0, m_1, \ldots, m_L)^\top$

- $\mathbf{Z}_t = (1, Z_{t,1}, \ldots, Z_{t,L})^\top$ is a multivariate time series

The model cont.

- The functions m_l reflect the time invariant structure of $\mathbf{Y}_t$
- $\widehat{m}_l$ is a nonparametric estimator of m_l obtained **directly** from the data points $X_{t,j}$, i.e. not from some estimated functions of $X_{t,j}$ as in PCA (piecewise constant, smoothed forward curves)
- The coefficients $Z_{t,l}$ describe the dynamic behavior of the forward curves
- The whole complex system can be modeled through a typical time series analysis of the estimates $\widehat{Z}_{t,l}$

Estimation

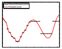
$$\mathbf{Z}_t^\top \mathbf{m}(\mathbf{X}_t) = \sum_{l=0}^{L} Z_{t,l} \sum_{k=1}^{K} a_{l,k}\psi_k(\mathbf{X}_t) = \mathbf{Z}_t^\top \mathbf{A}\psi(\mathbf{X}_t)$$

- $\psi(\cdot) = (\psi_1, \ldots, \psi_K)^\top$ is a vector of known expansion functions (e.g. B-splines)
- $\mathbf{A} \in \mathbb{R}^{(L+1)\times K}$ is a matrix of coefficients
- the smoothing parameters L (dimension of the time series; we use $L = 3, \ldots, 6$) and K (number of series expansion functions; $K = 19$ functions on 16 knots) have to be specified in advance

Estimation cont.

The least squares estimators $\widehat{\mathbf{Z}}_t = (\widehat{Z}_{t,0}, \ldots, \widehat{Z}_{t,L})^\top$ and $\widehat{\mathbf{A}} = \left(\widehat{a}_{l,k}\right)_{l=0,\ldots,L;k=1,\ldots,K}$ are obtained from

$$\sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - \widehat{\mathbf{Z}}_t^\top \widehat{\mathbf{A}} \psi(X_{t,j}) \right\}^2 = \min_{\widehat{\mathbf{Z}}_t, \widehat{\mathbf{A}}}$$



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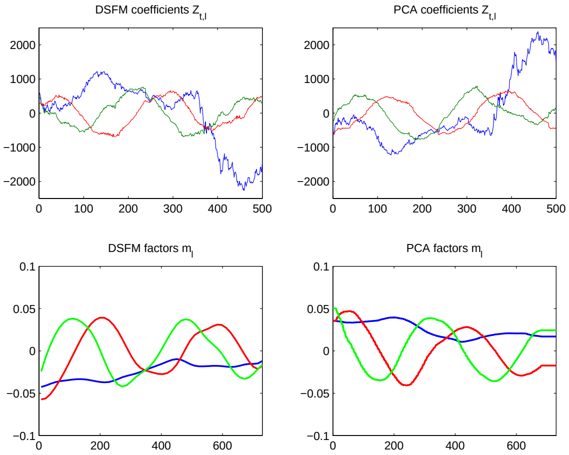
The data

- Database of Nord Pool futures and forward prices from the period Jan. 4, 1999 – May 23, 2002, i.e. 843 (business) days
- A 500 day window is used for calibration
- For each day, 1, 5, 10, 25 and 125 day-ahead forward curve forecasts is computed
- This leaves us with $800 - 500 - 125 = 218$ days for which the procedure (calibration + forecast) is repeated
- Both DSFM and PCA models for various L $(= 3, \ldots, 6)$ are evaluated

Sample DSMF and PCA fits ...



... and their dynamics

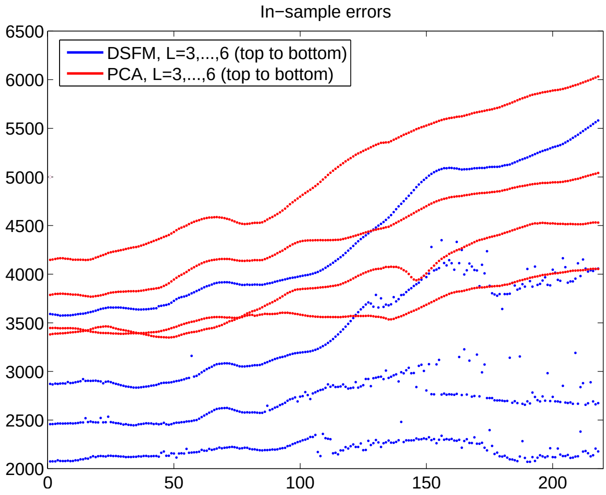
In-sample error measure

- We compute an absolute in-sample error weighted by the length of the delivery period for each contract

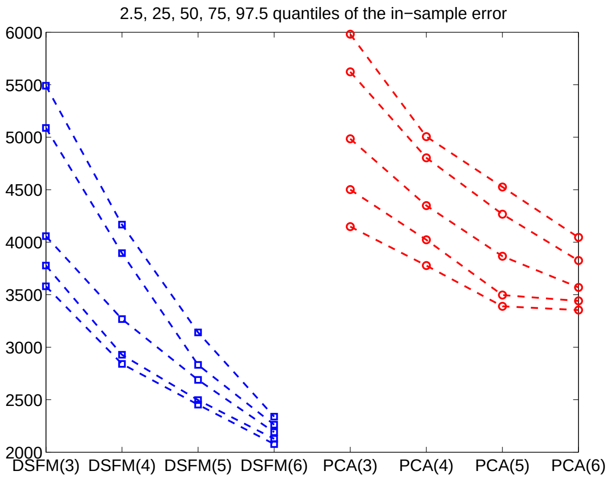
$$\epsilon_t = \sum_j |\text{Model}(X_{t,j}) - Y_{t,j}| \cdot ||I_{t,j}||$$

- where $X_{t,j}$ are the observed maturities (mid-points of the delivery periods) $j = 1, 2, \ldots$ for day t
- $Y_{t,j}$ are the respective prices
- $I_{t,j}$ are the respective time intervals with a constant price $Y_{t,j}$, such that $\bigcup_j I_{t,j} = (0, 2]$ years

In-sample errors



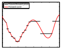
In-sample error statistics



Forecasting setup

- For a 500 day window the DSFM and PCA models (with $L = 3, \ldots, 6$ factors) are calibrated
- For $l = 2, \ldots, L$, $\widehat{Z}_{t,l}$ generally exhibit a seasonal pattern
- A sinusoidal function $g_l(t) = A_l \sin(B_l t + C_l)$ is fitted and removed yielding $\widetilde{Z}_{t,2}, \ldots, \widetilde{Z}_{t,L}$

$$
\begin{aligned}
Y_{t,j} &= m_0(X_{t,j}) + \sum_{l=1}^{L} \widehat{Z}_{t,l} m(X_{t,j}) + \varepsilon_{t,j} \\
&= m_0(X_{t,j}) + \sum_{l=2}^{L} g_l(t) m(X_{t,j}) + \sum_{l=1}^{L} \widetilde{Z}_{t,l} m(X_{t,j}) + \varepsilon_{t,j}
\end{aligned}
$$



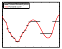
Forecasting models

- Random Walk (**RW**): forward (futures) prices from day t

$$Y^*_{t+h,j} = \widehat{m}_0(X_{t+h,j}) + \sum_{l=1}^{L} \widehat{Z}_{t,l}\widehat{m}_l(X_{t+h,j}) + \varepsilon_t$$

- Trend update (**STr** for DSFM and **PTr** for PCA): the sinusoidal trend g_l is forecasted for $l = 2, \dots, L$ and added to the forward price forecast

$$Y^*_{t+h,j} = \widehat{m}_0(X_{t+h,j}) + \sum_{l=2}^{L} g_l(t+h)\widehat{m}_l(X_{t+h,j}) + \sum_{l=1}^{L} \widetilde{Z}_{t,l}\widehat{m}_l(X_{t+h,j}) + \varepsilon_t$$



Forecasting models cont.

- Trend update from model (**STr2**, **PTr2**): Like 'Trend update', but without the error term

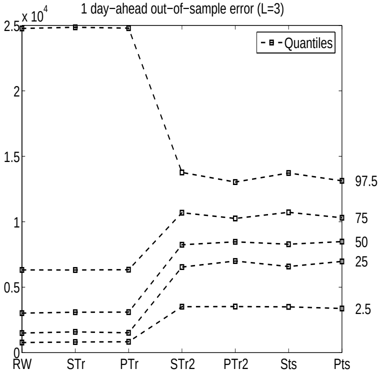
$$Y^*_{t+h,j} = \widehat{m}_0(X_{t+h,j}) + \sum_{l=2}^{L} g_l(t+h)\widehat{m}_l(X_{t+h,j}) + \sum_{l=1}^{L} \widetilde{Z}_{t,l}\widehat{m}_l(X_{t+h,j})$$

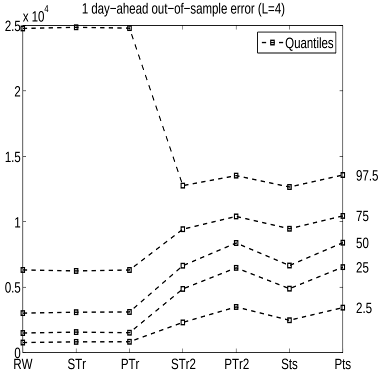
Forecasting models cont.

- Full forecast with AR(2) time series (**Sts**, **Pts**): Additionally includes AR(2) forecasts of all coefficient time series $\widehat{Z}_{t,1}, \widetilde{Z}_{t,2}, \ldots, \widetilde{Z}_{t,L}$

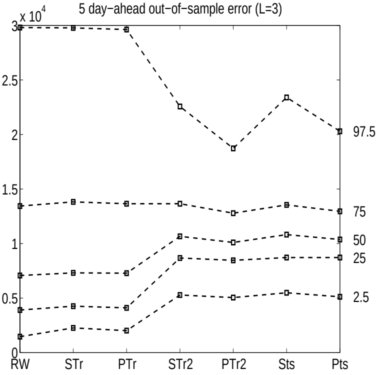
$$Y^*_{t+h,j} = \widehat{m}_0(X_{t+h,j}) + \sum_{l=2}^{L} g_l(t+h)\widehat{m}_l(X_{t+h,j}) + \sum_{l=1}^{L} \widetilde{Z}^*_{t+h,l}\widehat{m}_l(X_{t+h,j})$$

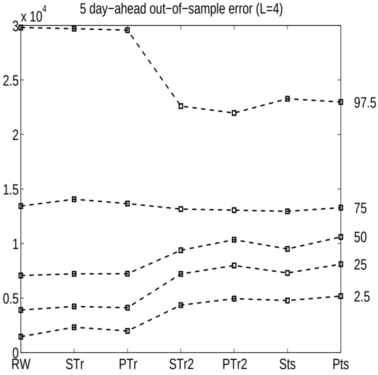
One day-ahead forecasts: $L = 3$ vs. $L = 4$



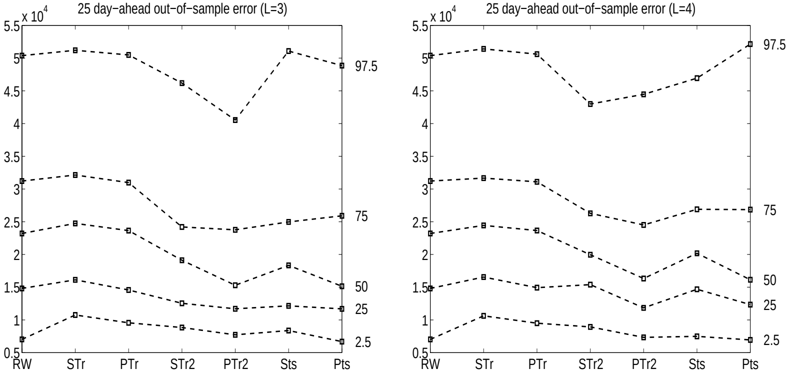


5 day-ahead forecasts: $L = 3$ vs. $L = 4$

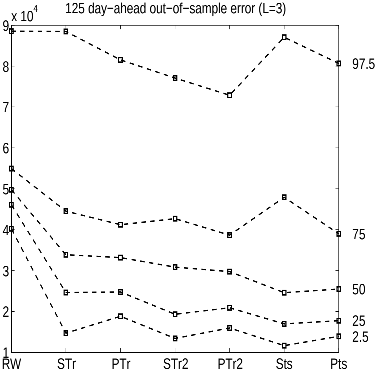


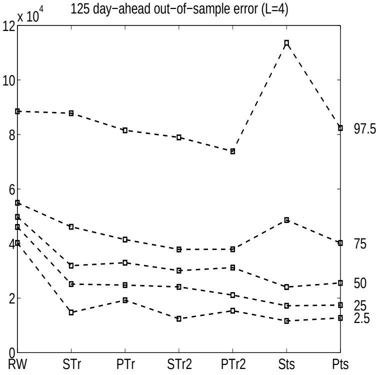


25 day-ahead forecasts: $L = 3$ vs. $L = 4$

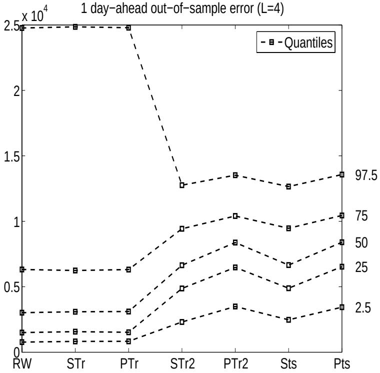


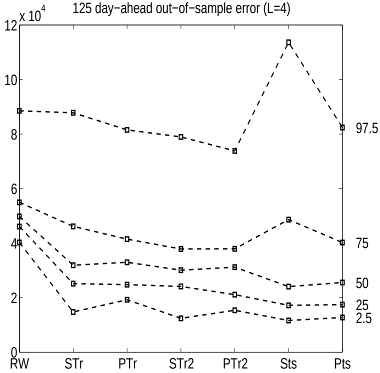
125 day-ahead forecasts: $L = 3$ vs. $L = 4$





Short vs. long term forecasts ($L = 4$)





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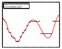
Conclusions

- The electricity forward curve is a complex object with a non-trivial structure
- Like PCA, DSFM allows for dimension reduction
- It differs in that
  - the fits are obtained in the **local neighborhood** of forward price–maturity pairs for a given day
  - curve estimation and dimension reduction is achieved in **one single step**
- DSFM offers **superior** in-sample performance

Conclusions cont.

- There is no clear winner in the out-of-sample forecasts
- DSFM-based models are slightly **better** than their PCA-based counterparts for short and long term forecasts and **worse** for medium term predictions
- The current forward curve (RW model) is on average the **best** predictor of the curve in the next few days, it is **inferior** for medium and long term forecasts
- Larger number of basis functions (larger L) **improves** short term forecasts but leads to **increased variance** of the longer term forecasts



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Fengler, M., Härdle, W., Mammen, E. (2007)
A semiparametric factor model for implied volatility surface dynamics, *Journal of Financial Econometrics* 5(2): 189–218.
See also: SFB 649 Discussion Paper 2005-020.

Borak, S., Härdle, W., Mammen, E., Park, B.U. (2007)
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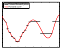

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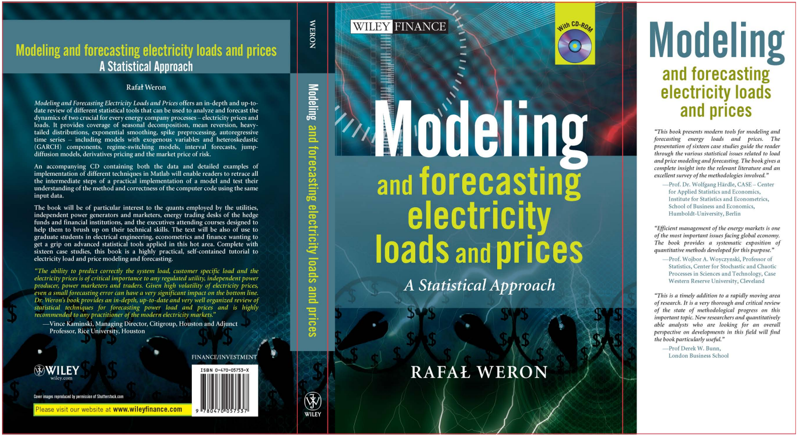
Modeling and forecasting electricity loads and prices
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