Markov regime-switching models for electricity prices

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Motivation

- Risk management and derivatives pricing often require a model for spot prices that is:

- Realistic
  - Why would we want an unrealistic model?!
- Parsimonious
  - Faster simulation, smaller calibration errors
- Statistically sound
  - We can calibrate any model to any dataset …
  - … but does it **really** fit the data? Does it **make sense**?

Agenda

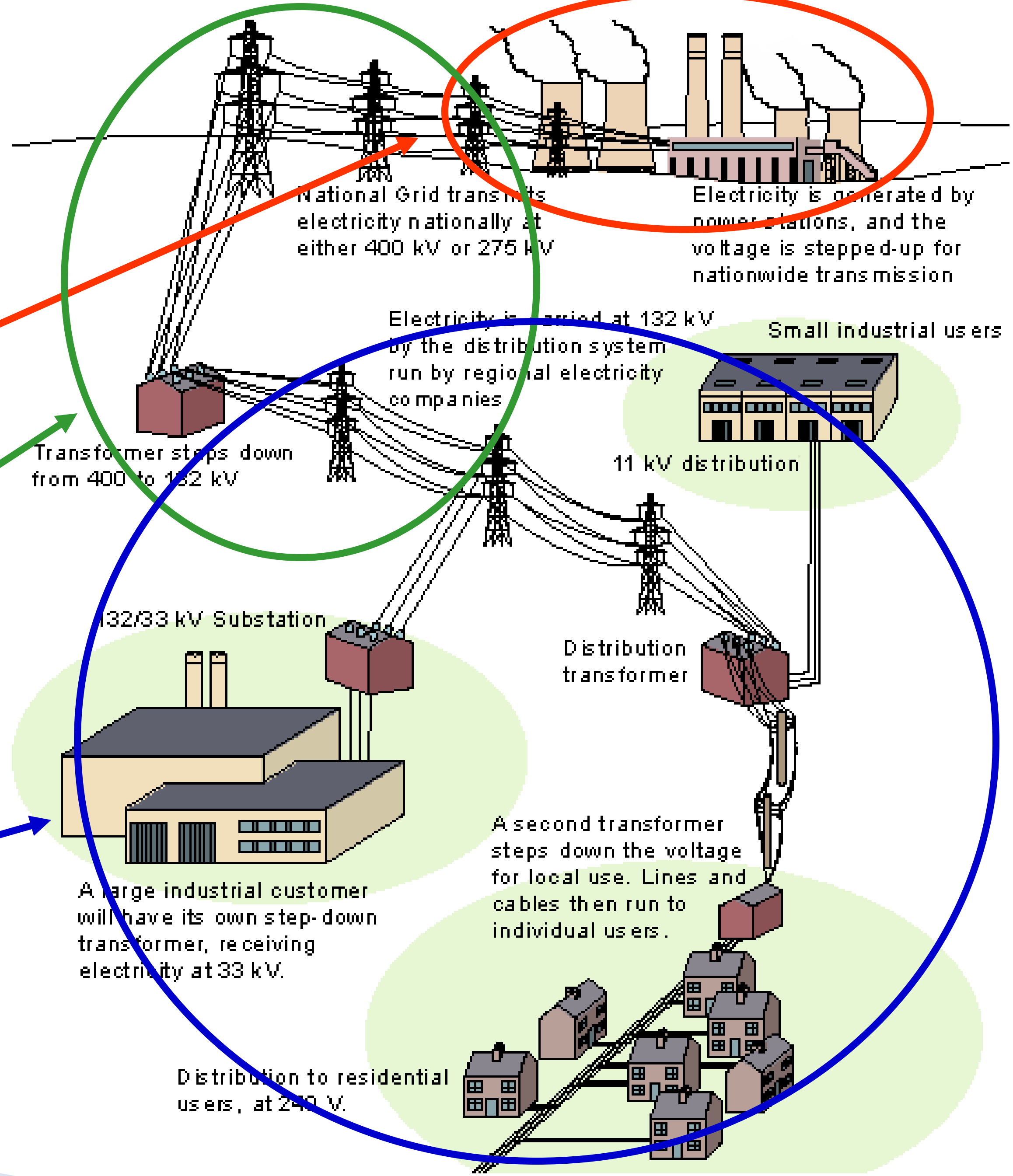
- Motivation
- **Power markets in a nutshell**
- Dealing with seasonality
- Case study: MRS models for the spot price

The power system

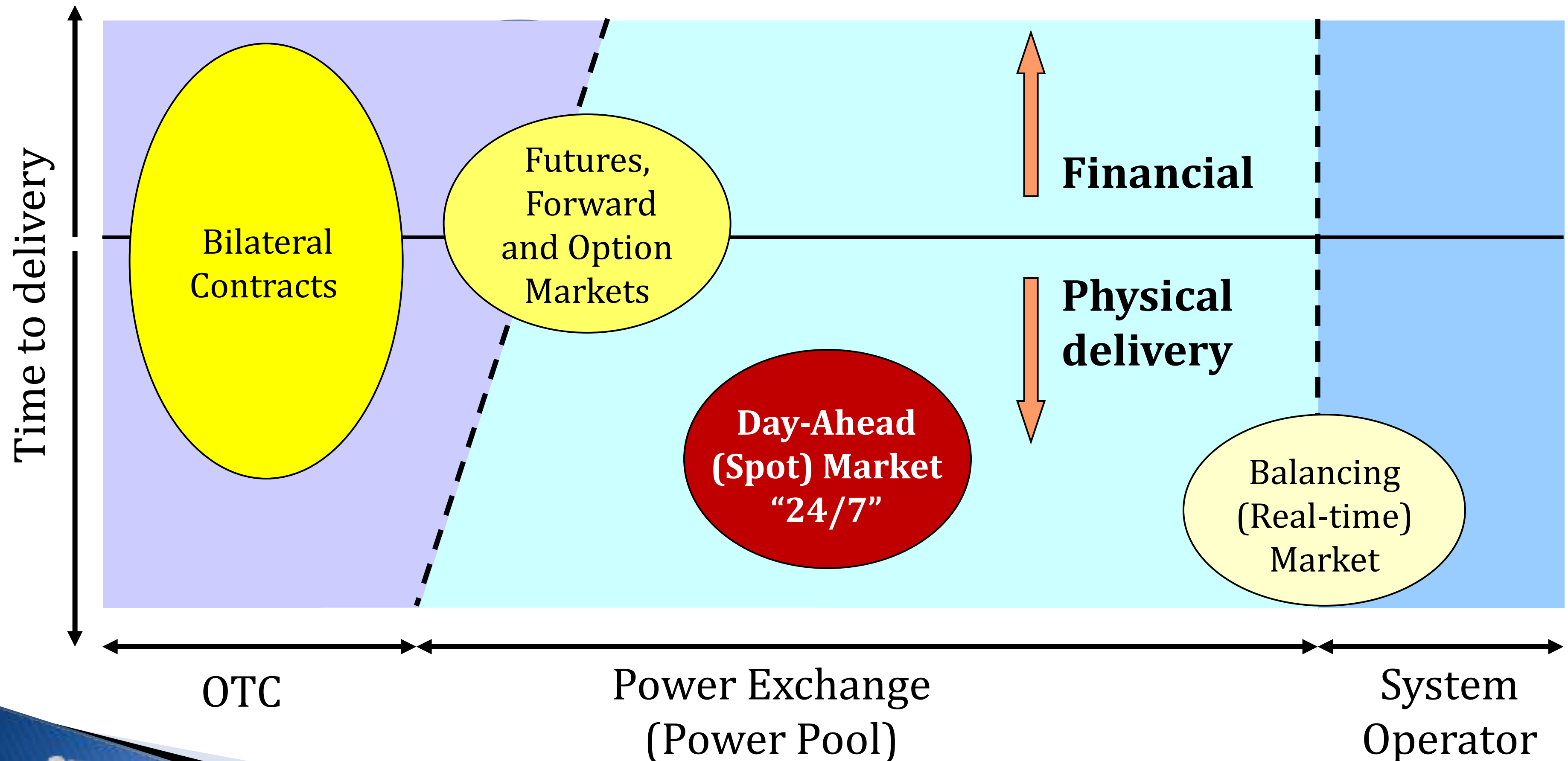
Generation

Transmission

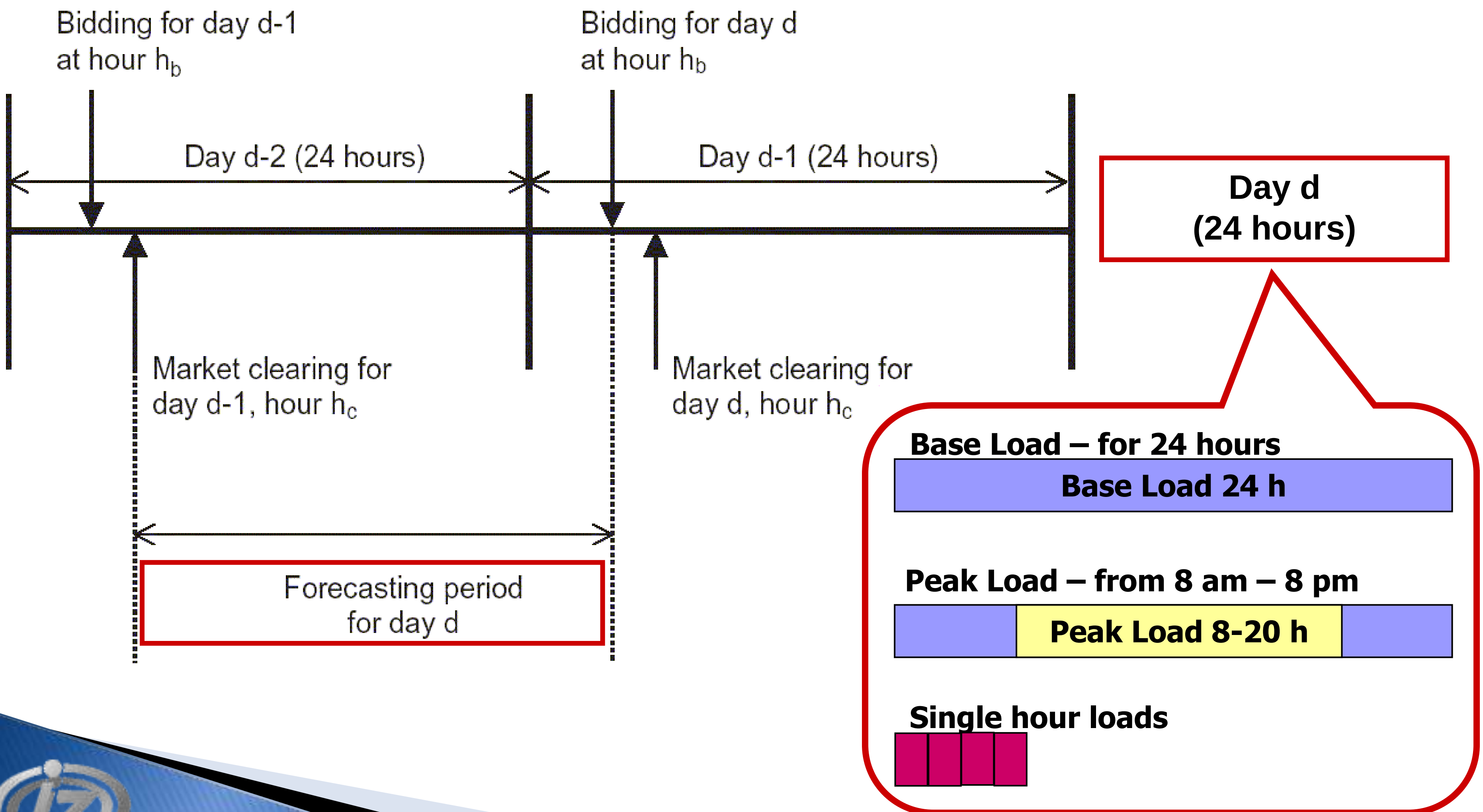
Retail



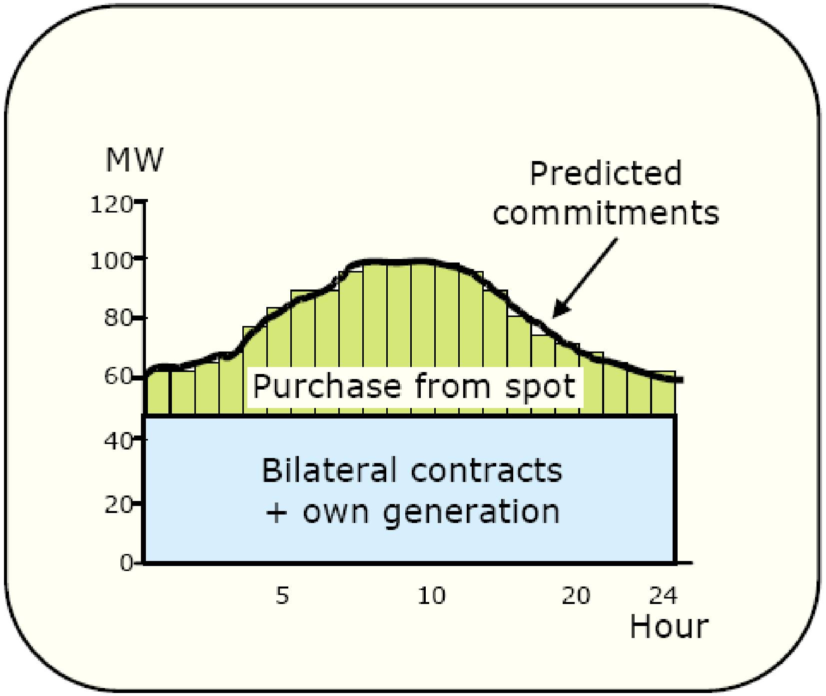
Wholesale electricity market structure

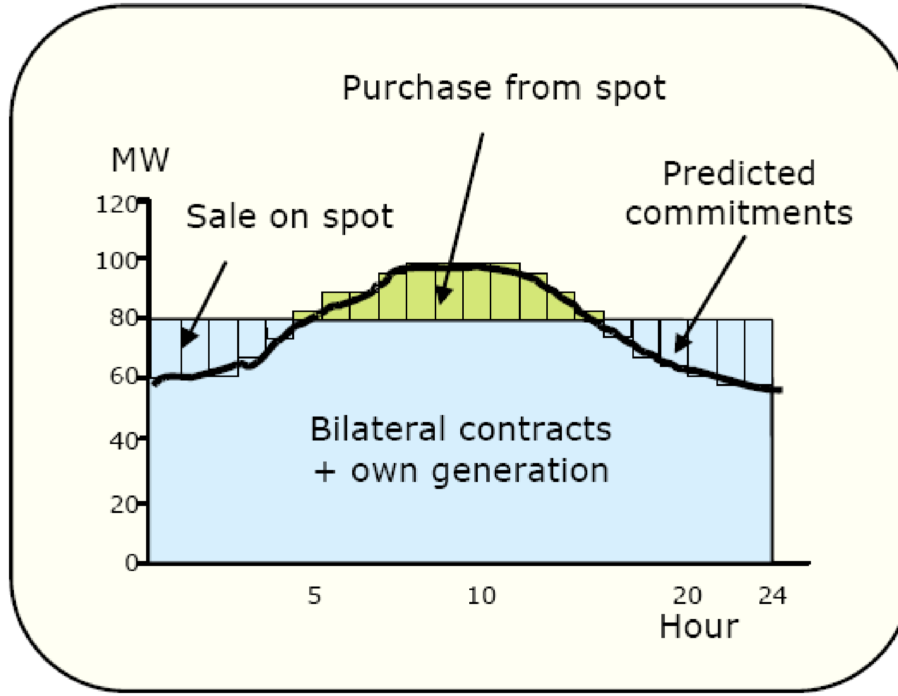


The spot

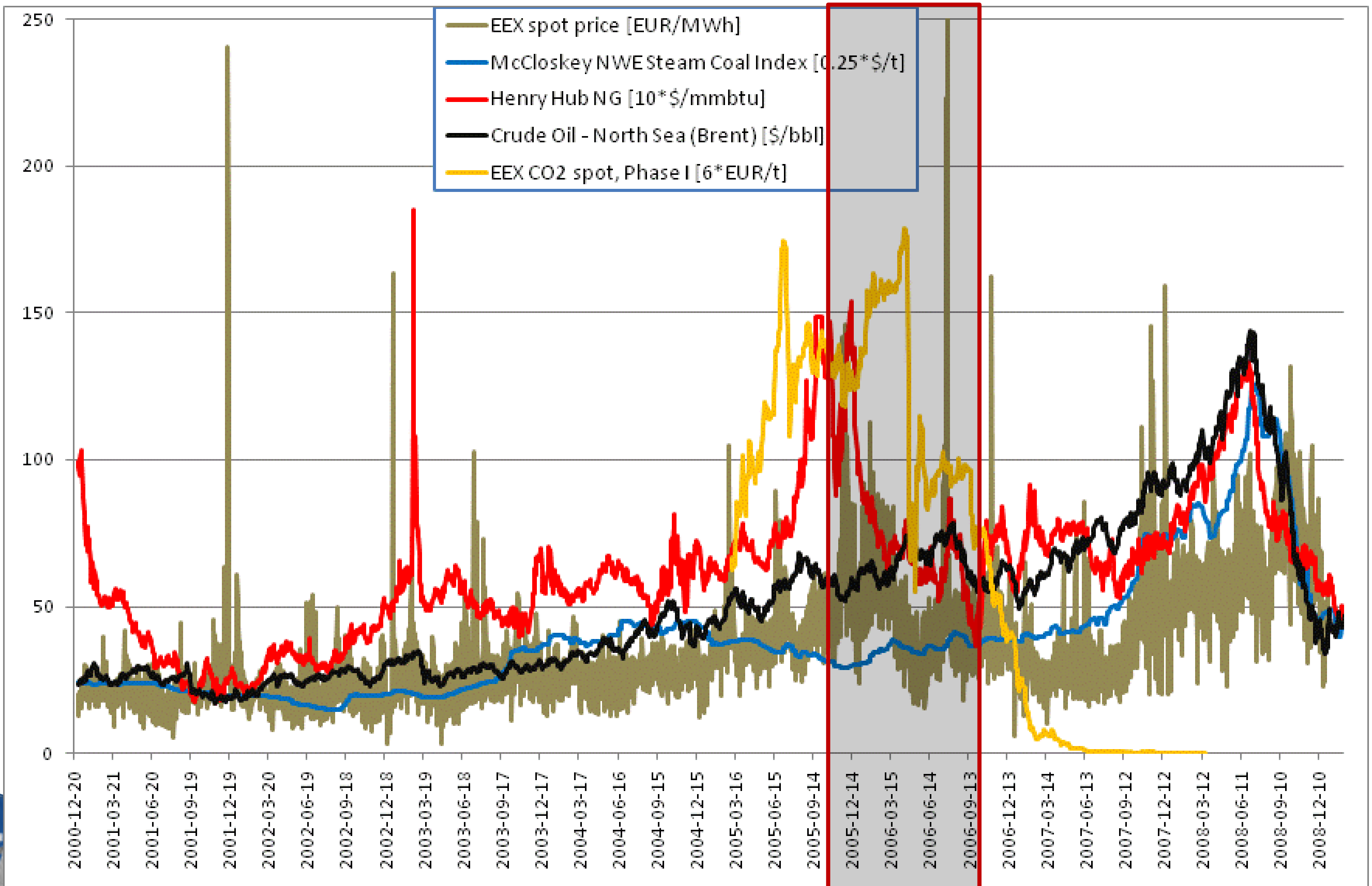

Bidding for day d-1
at hour h_b
Bidding for day d
at hour h_b
Day d-2 (24 hours)
Day d-1 (24 hours)
Day d
(24 hours)
Market clearing for
day d-1, hour h_c
Market clearing for
day d, hour h_c
Forecasting period
for day d
Base Load – for 24 hours
Base Load 24 h
Peak Load – from 8 am – 8 pm
Peak Load 8-20 h
Single hour loads

Use of the spot market

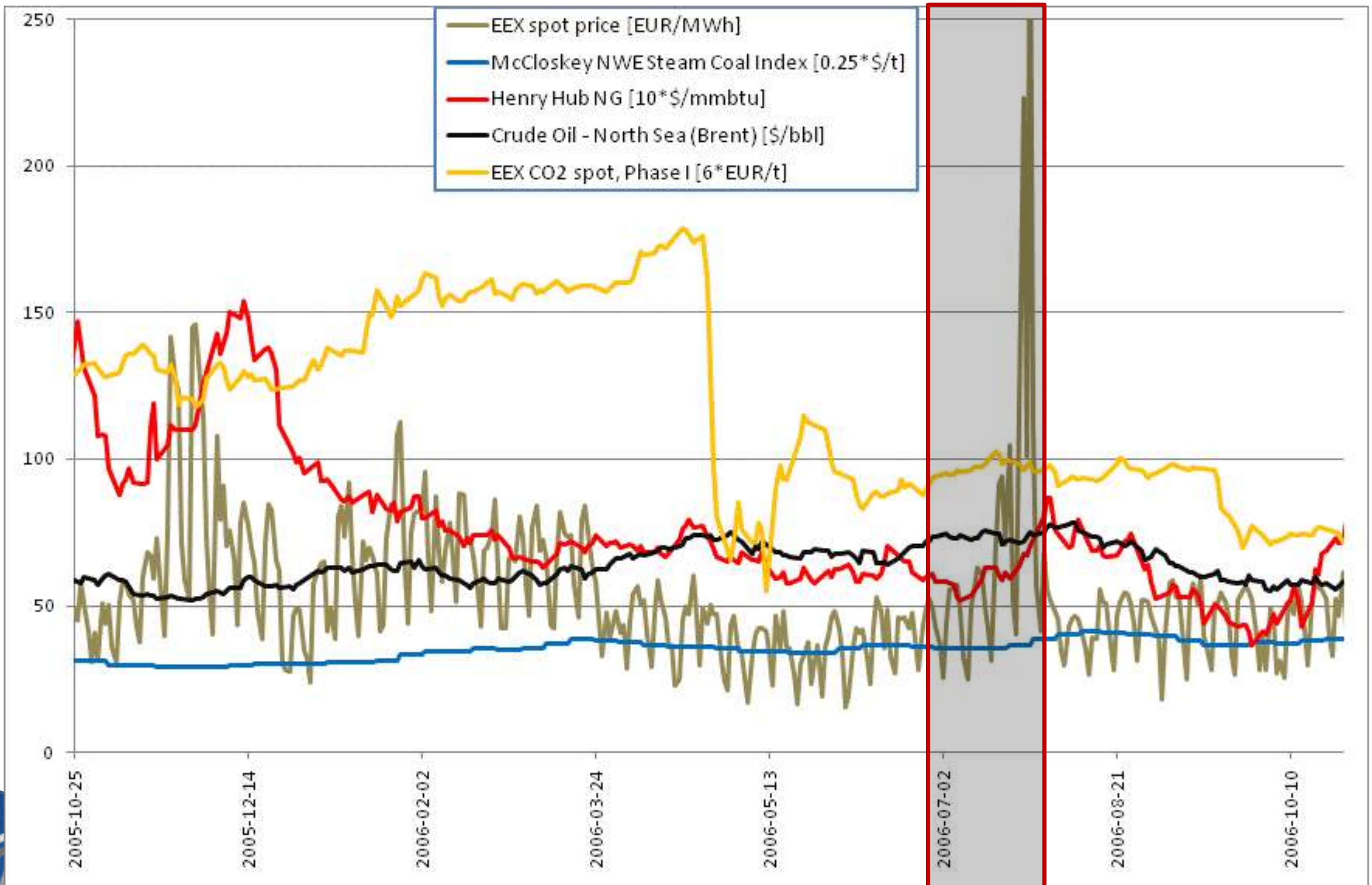




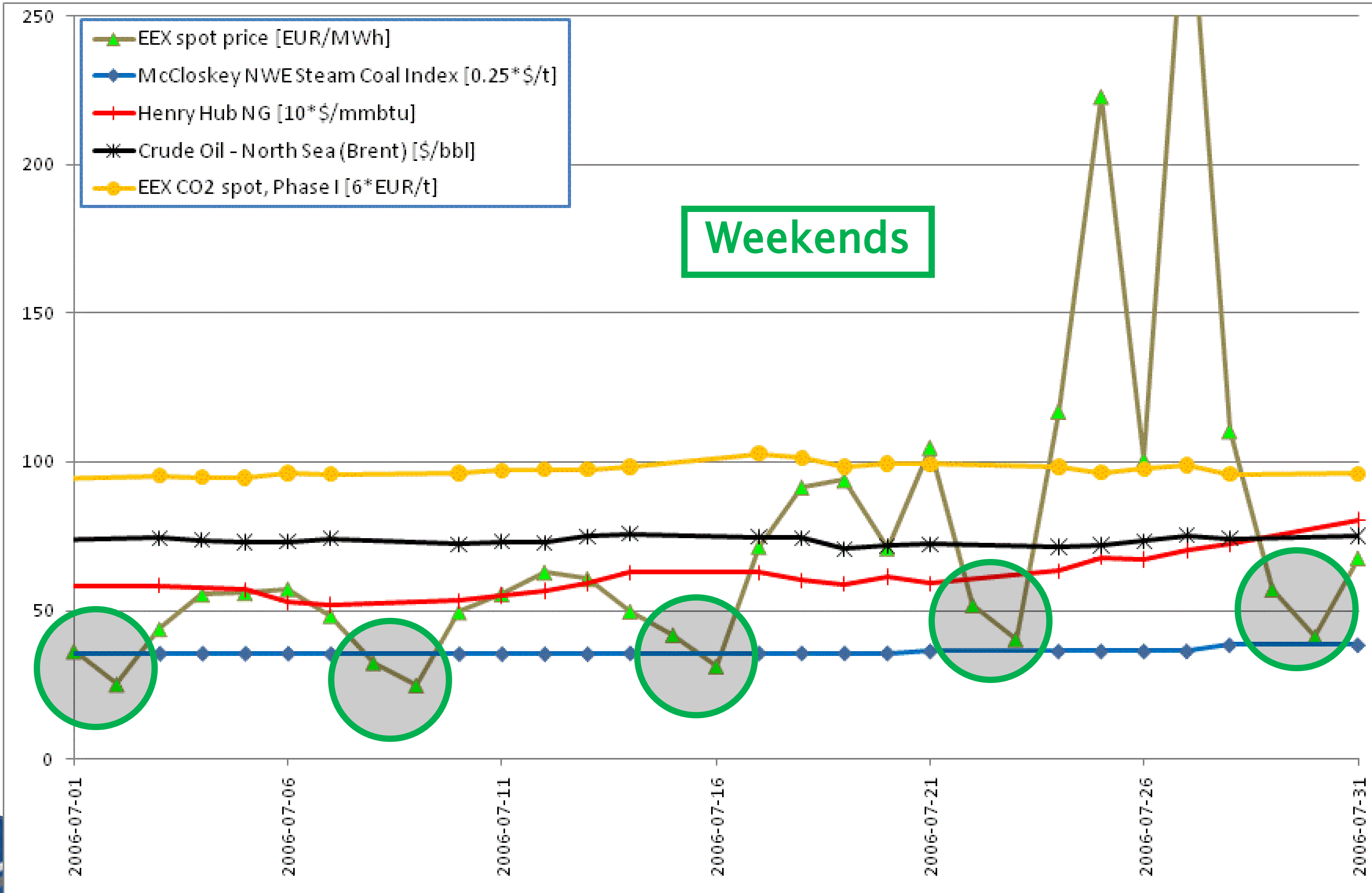
Energy commodities



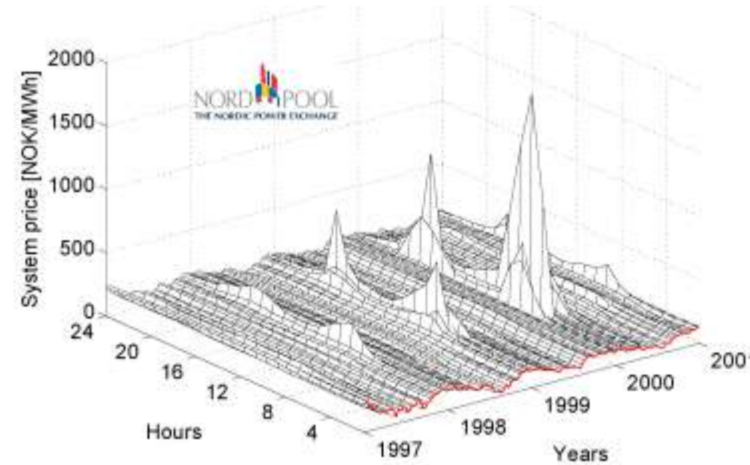
Zoom in (one year)



Zoom in (one month)

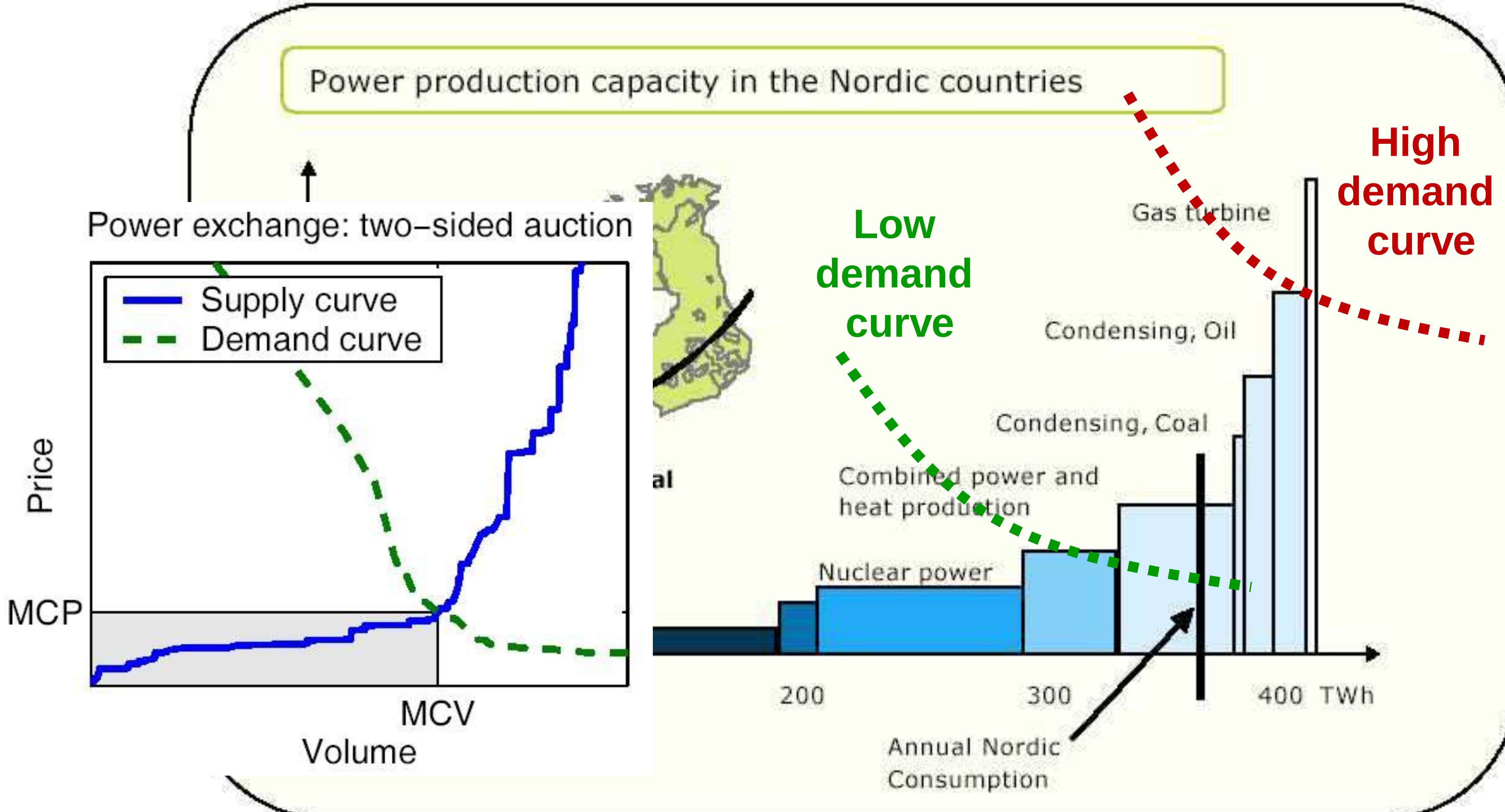


Electricity is a (special) commodity

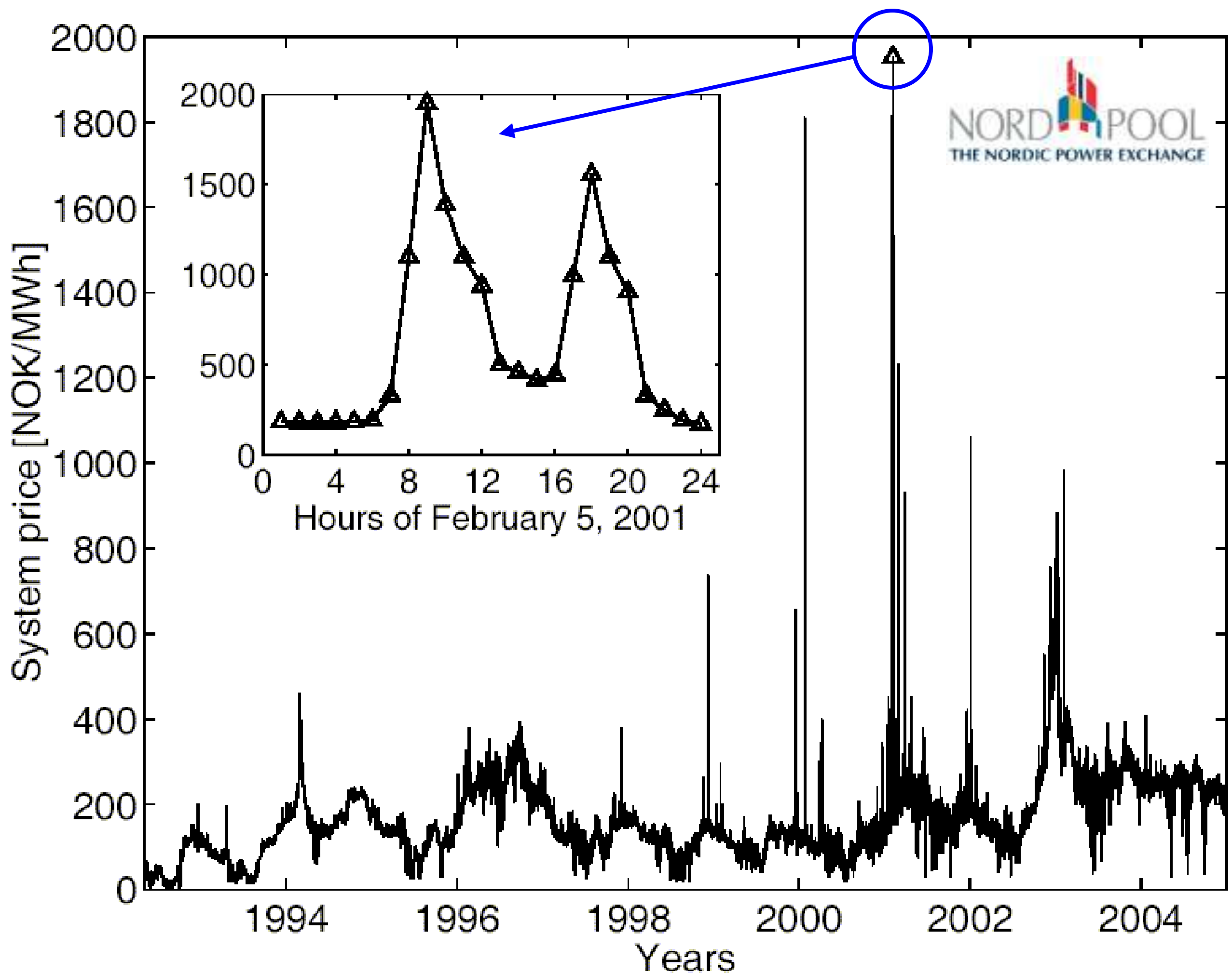


- Seasonality
  - Daily, weekly, annual
- Weather dependency
- Non- or limited storability
- Transmission constraints
- Spikes in prices and loads (consumption)
  - Extreme volatility, up to 50% for daily returns
- "Inverse leverage effect"
  - Prices and volatility are positively correlated
  - Both are negatively related to the inventory level
- "Samuelson effect"
  - Volatility of forward prices decreases with maturity

Supply stack and the spikes



Price spikes … are transient



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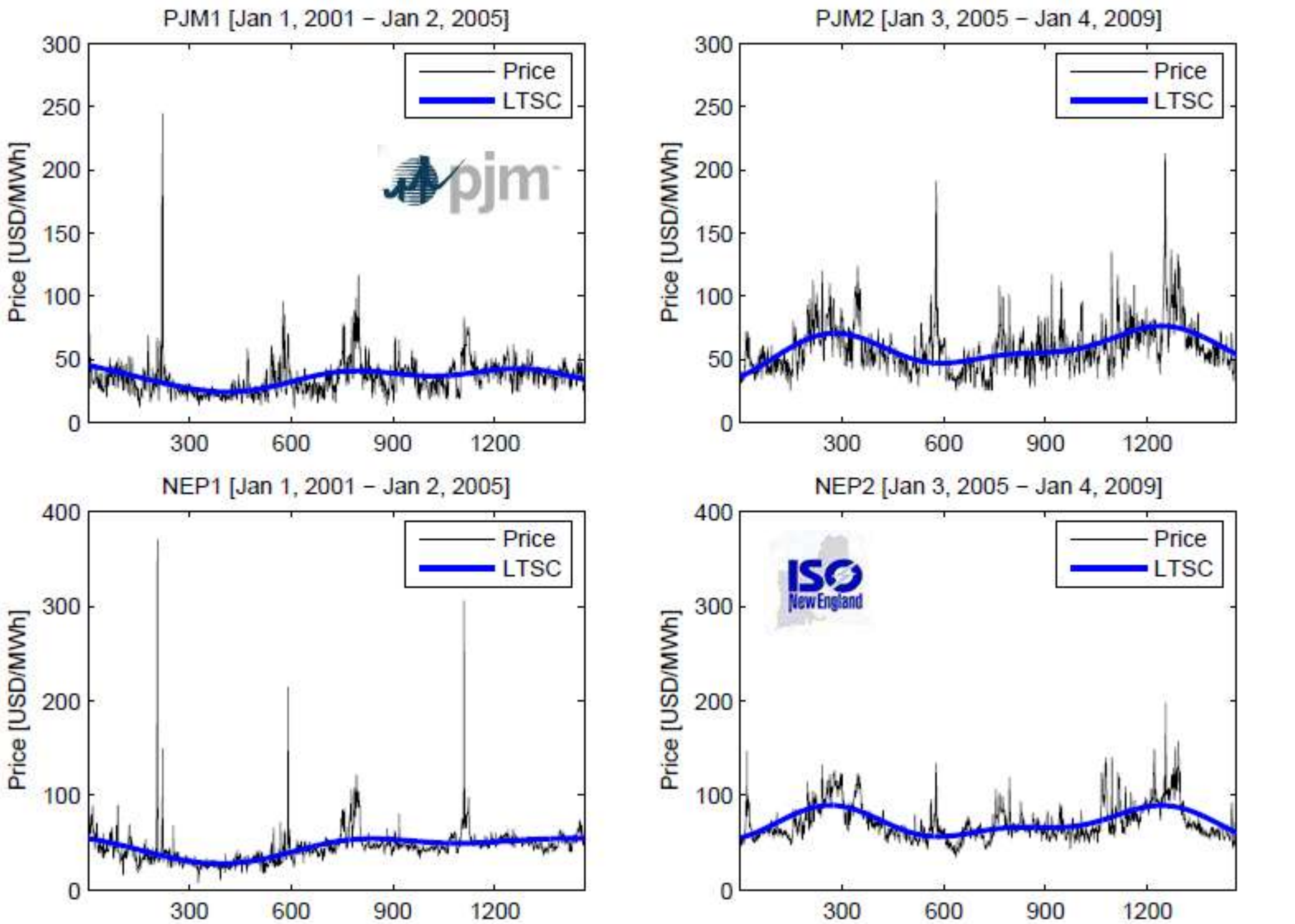
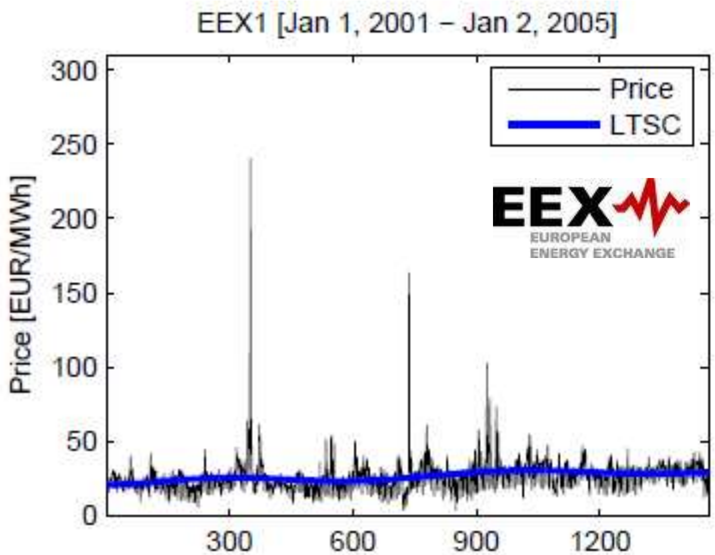
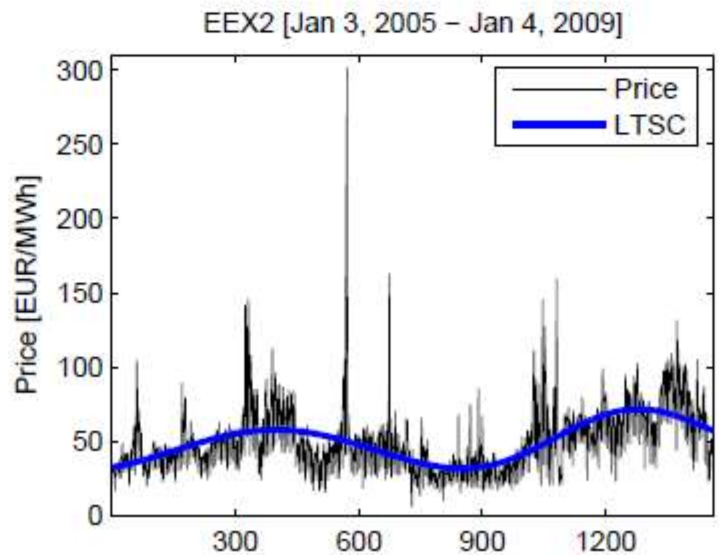
- Power markets in a nutshell
- **Dealing with seasonality**
- Case study: MRS models for the spot price

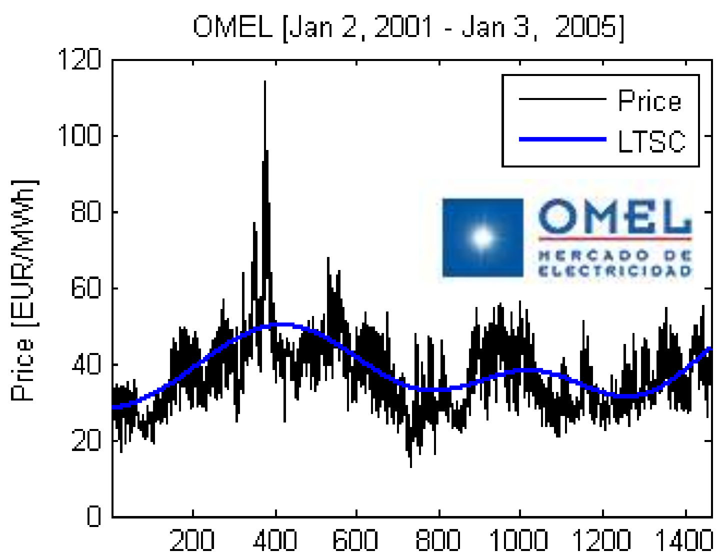
The datasets: two U.S. markets
(4 year samples – 1463 observations)



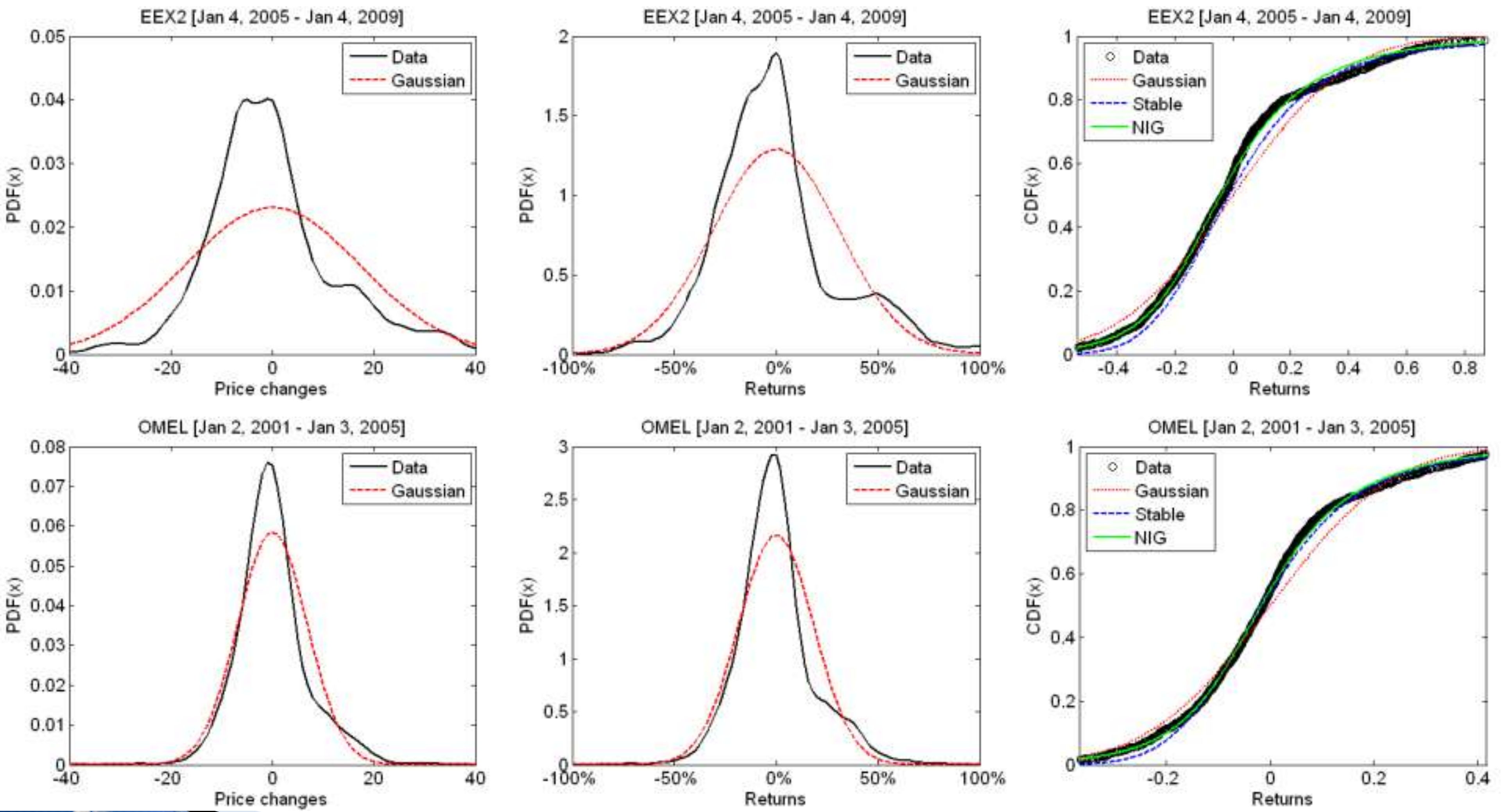
… and two European
(4 year samples – 1463 observations)







The need to deseasonalize …



Dealing with seasonality

- Spot price P_t (denoted also by S_t) is typically modeled as a sum (or a product) of
  - A 'deterministic' (seasonal) component Λ_t and
  - A purely stochastic component X_t
- A seasonal model for Λ_t is usually calibrated and removed from the data prior to estimating X_t
  - Extreme observations may impact the estimate of Λ_t

- Possible solutions
  - Use preprocessing – detect & replace price spikes with more 'normal' values; add them back before estimating X_t
  - Use methods 'immune' to outliers (quantiles, wavelets)

Short-term seasonality

- Differencing
  - Simplest form: $X_t = P_t - P_{t-7}$
- Removing the mean or median week

  Mo Tu We Th Fr Sa Su (week #1)

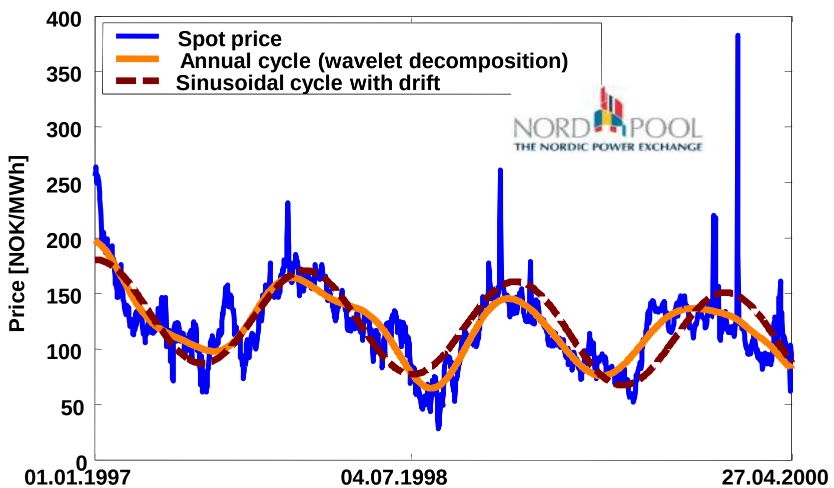
  Mo Tu We Th Fr Sa Su (week #2)

  ... and holidays

  Mo Tu We Th Fr Sa Su **Ho** (week #1)
- Moving average (MA) method
  - Calculate $m_t = (P_{t-3} + ... + P_{t+3})/7$
  - Subtract the mean of deviations $(P_{k+7j} - m_{k+7j})$ from P_t

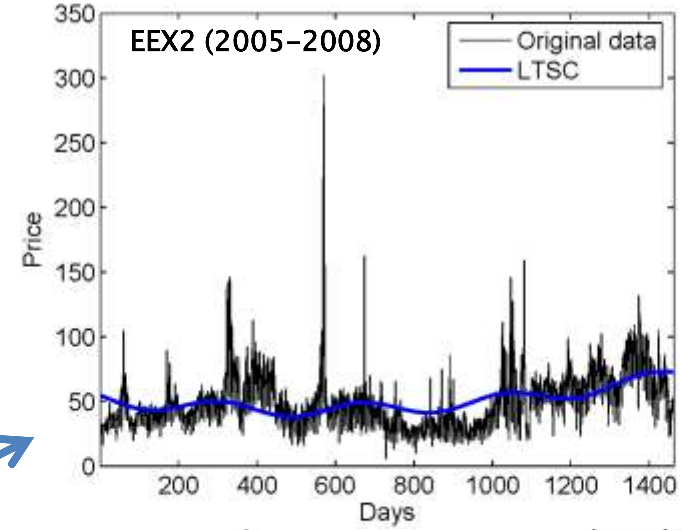
Long-term seasonality

- Fitting piecewise constant functions (dummy variables) for each month
    - Bhanot (2000), Haldrup & Nielsen (2006), Knittel & Roberts (2005), Lucia & Schwartz (2002)
    - For each day of the week → corresponds to mean/median week or moving average method
- Fitting (a sum of) **sinusoids** with trend
    - Bierbrauer et al. (2007), Borovkova & Permana (2006), Cartea & Figueroa (2005), De Jong (2006), Geman & Roncoroni (2006), Lucia & Schwartz (2002), Pilipovic (1997), Weron (2006)
- **Wavelet smoothing**
    - Weron et al. (2004), Trück et al. (2007), Weron (2008), Janczura & Weron (2009)

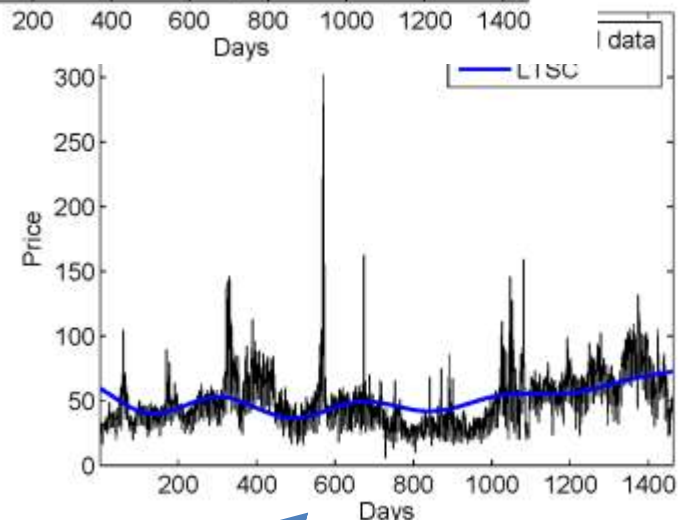


Fitting sinusoids

- Sinusoid with linear trend
  - $\Lambda_t = A\cdot\sin(2\pi(t+B)) + C + D\cdot t$
  - t – time in years



- … with cubic trend
  - $\Lambda_t = A\cdot\sin(2\pi(t+B)) + C + D\cdot t + E\cdot t^2$
- … with linear trend and linear amplitude
  - $\Lambda_t = (A + F\cdot t)\cdot\sin(2\pi(t+B)) + C + D\cdot t$



- … with cubic trend and linear amplitude
  - $\Lambda_t = (A + F\cdot t)\cdot\sin(2\pi(t+B)) + C + D\cdot t + E\cdot t^2$

Wavelet smoothing 1/2

- Any signal (here: the spot price) can be built up as a sequence of projections onto
  - one father wavelet S_J and a sequence of mother wavelets $\{D_j\}$

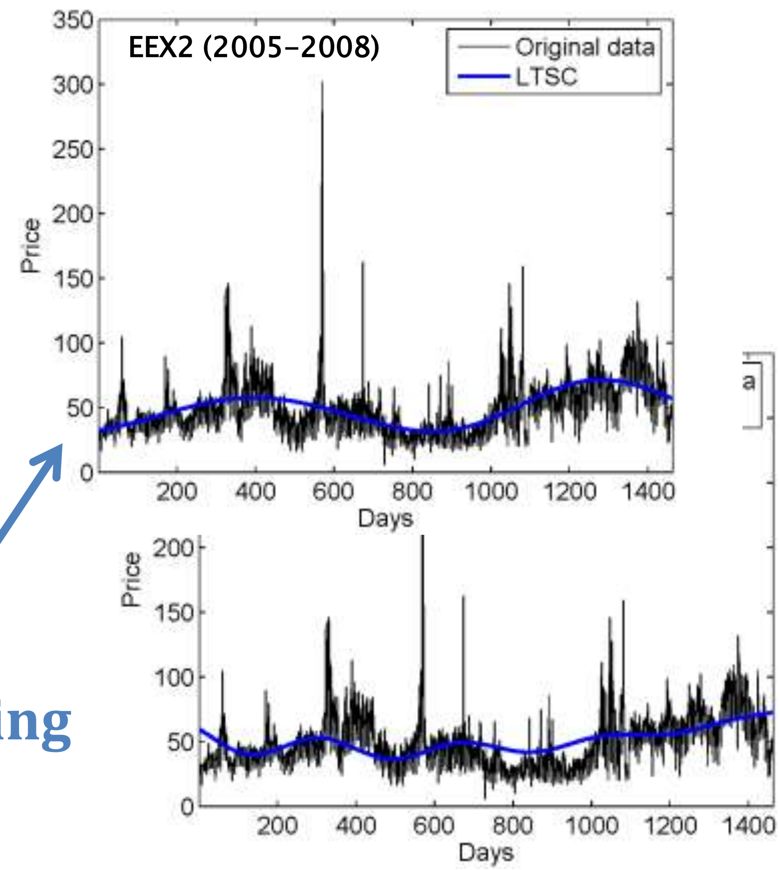
$$x(t) = S_J + D_J + D_{J-1} + \ldots + D_1$$

  - where 2^J is the maximum scale sustainable by the number of observations

  - **Sines** are localized in frequency (characteristic scale)
  - **Wavelets** are also localized in time (space)

Wavelet smoothing 2/2

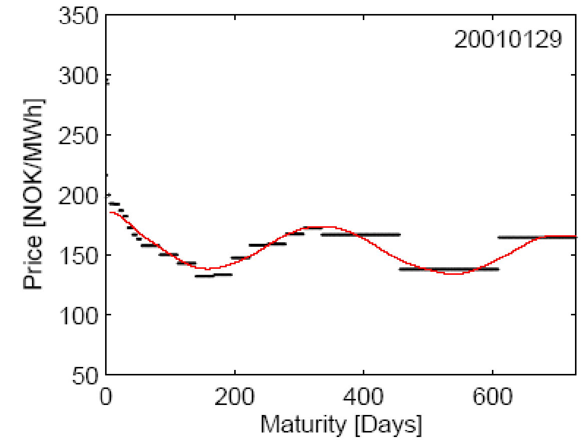
- At the coarsest scale the signal can be estimated by S_J
  - By adding a mother wavelet D_j of a lower scale $j = J-1, J-2, \ldots$, we obtain a better estimate of the original signal $\rightarrow$ **lowpass filtering**

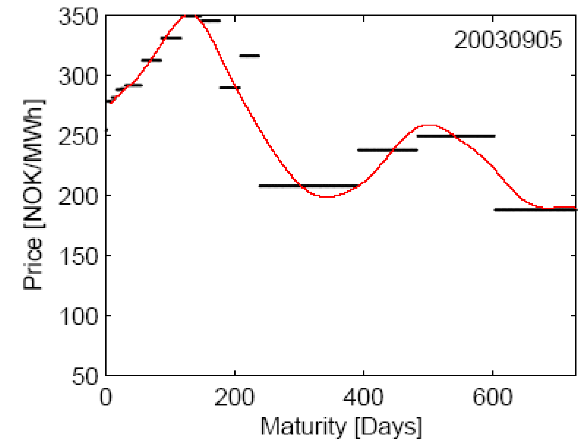


- For daily data the S_3, S_5 and S_8 approximations roughly correspond to
  - weekly ($2^3 = 8$ days),
  - monthly ($2^5 = 32$ days) and
  - **annual ($2^8 = 256$ days) smoothing**

Forecasting long-term seasonality

- Piecewise constant functions
  - What about trends?
- Sinusoids
  - How to predict the trend?
  - Model risk
- Wavelets
  - Extrapolate the smoothed price?
- Forward prices
  - Smooth interpolation of forward prices
  - What about the far end?
  - What about risk premia?

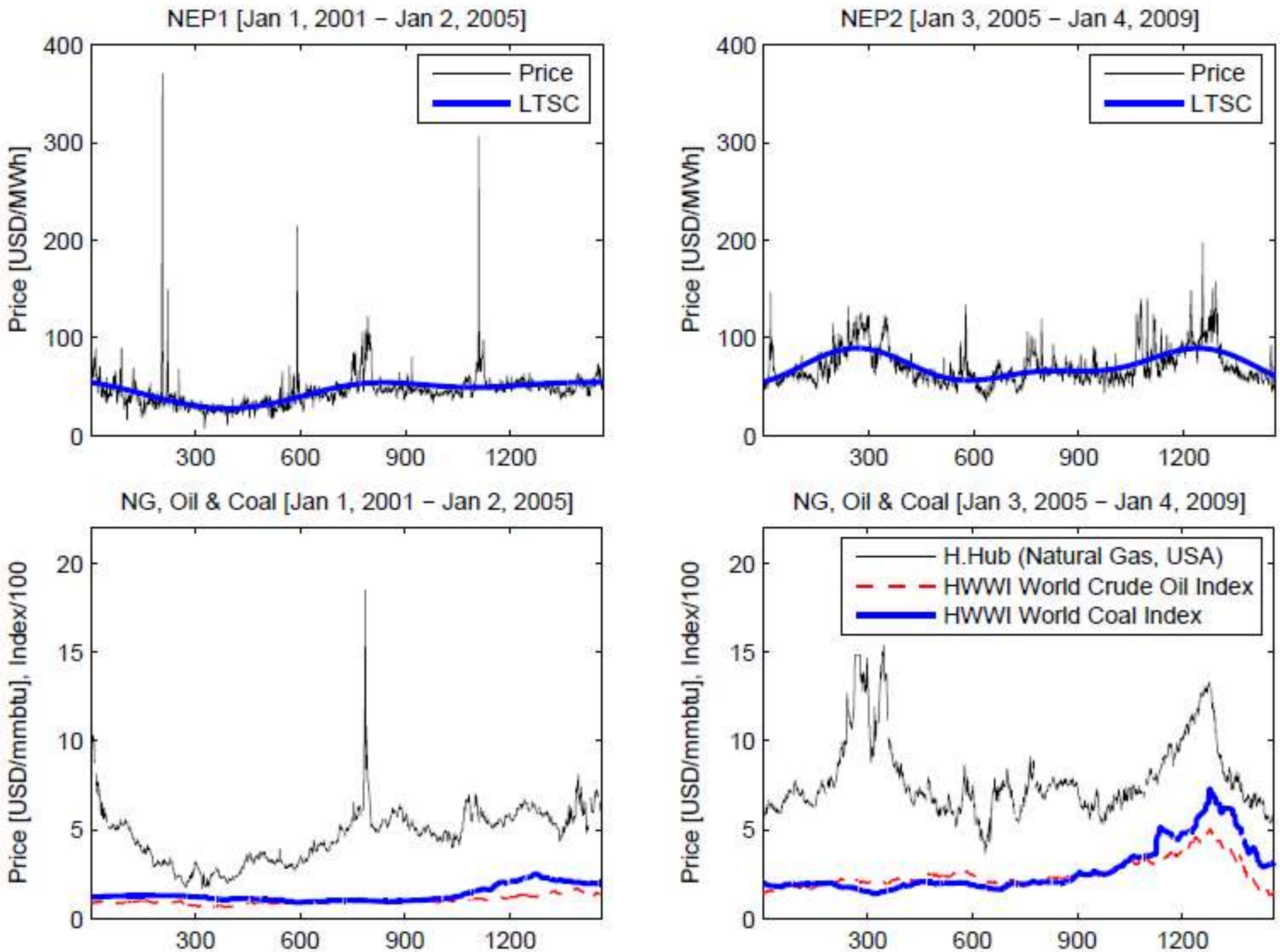




Seasonal decomposition

- Let the seasonal component Λ_t be composed of
  - **A long-term seasonal trend T_t**
    - Changing climate/consumption conditions throughout the year and the long-term non-periodic structural changes (fuel prices)
  - And a weekly periodic part s_t
- First, T_t is estimated from daily spot prices P_t
  - Using 'annual' wavelet smoothing (J=8)
- Next, s_t is removed by applying the MA method
- Finally, the deseasonalized prices, i.e. $P_t - T_t - s_t$, are shifted: min(new process) = min(P_t)

Rationale for the wavelet smoother



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Reduced form models for the spot price

- Typically the deseasonalized spot electricity price X_t is assumed to follow some kind of a jump-diffusion (JD) process:

$$dX = \mu(X,t)dt + \sigma(X,t)dB + dq(X,t)$$

Generalized drift (possibly mean reverting)

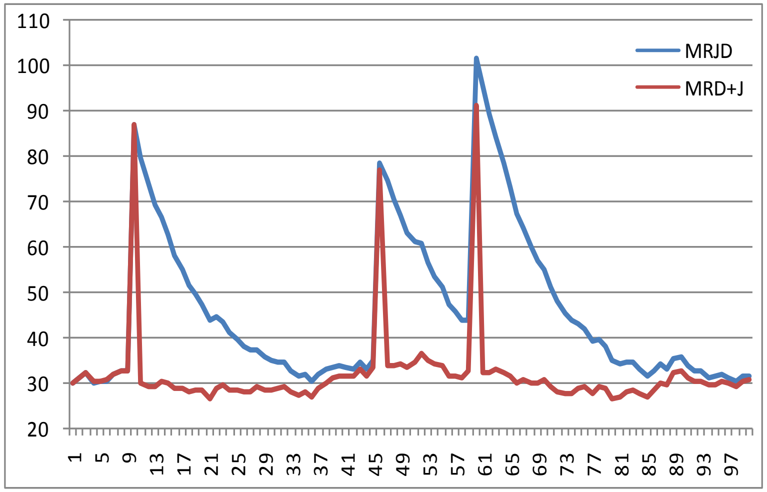
Generalized volatility (possibly heteroskedastic)

Pure jump process with given intensity and severity – e.g. a compound Poisson process: $dq(X,t) = XdN_t$

- Clewlow & Strickland, 2000; Eydeland & Geman, 2000; Kaminski, 1999

Problems with JD models

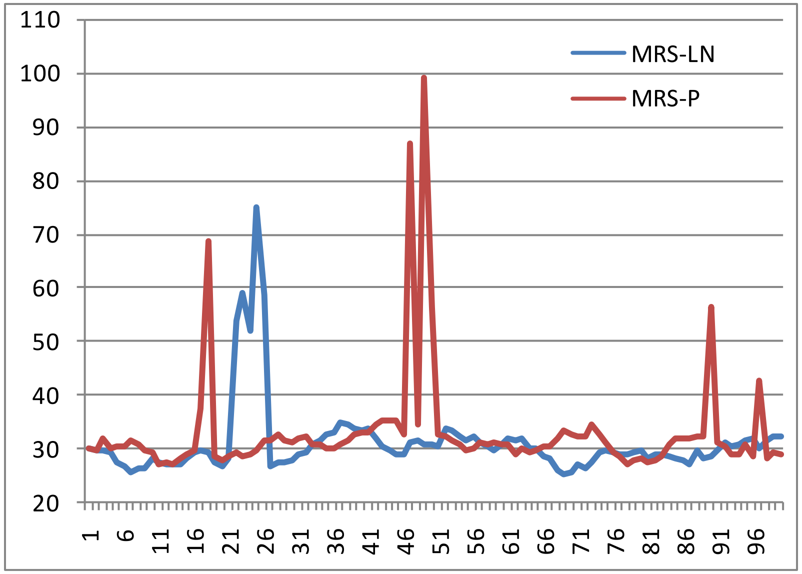
- After a jump the price is forced back to its normal level
  - by mean reversion (**MRJD**)
  - by mean reversion coupled with downward jumps
    - Deng 1999; Escribano et al., 2002; Geman & Roncoroni, 2006
  - by a combination of mean reversions with different rates
    - Benth et al., 2007



- Alternatively, a positive jump may be always followed by a negative jump of approximately the same size – especially on the daily scale (**MRD+J**)
    - Weron et al., 2004; Weron, 2008

Problems with JD models cont.

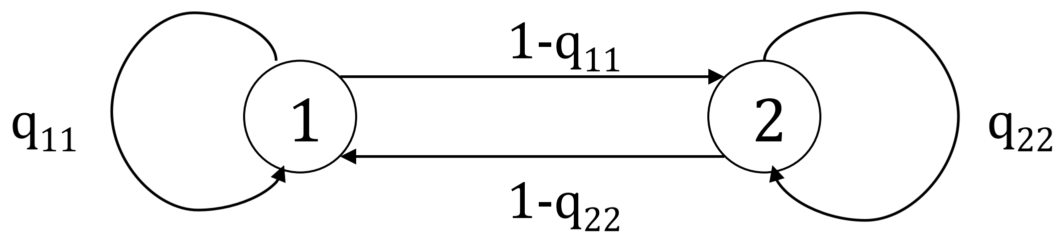
- What about periods of consecutive jumps?
  - Grid congestion, outage
- Solution:
  - Allow the process to ‘stay’ in the ‘jump regime’ with some probability
  - **Regime-switching models**



Markov regime switching (MRS) models

- Assume that the switching mechanism can be governed by a latent random variable that follows a Markov chain with two (or more) possible states

  A two-state regime model: X_t = {1,2}



  Transition probabilities:

$$\mathbf{Q} = (q_{ij}) = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = \begin{pmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{pmatrix}$$

- The regimes are only latent, not directly observable
- Estimation via EM (expectation–maximization) algorithm

(Dempster et al., 1977; Hamilton, 1990; Kim, 1993)

First generation MRS models for electricity spot log–prices

- Ethier & Mount (1998) proposed a model with 2 regimes governed by AR(1) processes
  - Concluded that there was strong support for the existence of different means and variances in the two regimes
- Huisman & de Jong (2003) proposed a 2-regime model
  - With a stable, mean-reverting AR(1) regime and
  - An independent spike (IS) regime modeled by a normal variable with a higher mean and variance
  - Bierbrauer et al. (2004), Weron et al. (2004) used log-normal and Pareto distributed spike regimes
  - De Jong (2006) introduced autoregressive, Poisson driven spike regime dynamics

Problems with first generation MRS models for log-prices

- Some authors reported that the 'expected spike sizes' ($\equiv E(Y_{t,\text{spike}}) - E(Y_{t,\text{base}})$) were **negative**
  - See e.g. De Jong (2006), Bierbrauer et al. (2007)
  - … but were not considered as evidence for model misspecification
- Regime classification was not checked but …
  - … the calibration scheme generally assigns all extreme prices to the spike regime
    - The 'sudden drops' in the log-price are not that interesting for price modeling and derivatives valuation
    - They appear extreme only because of the log transform

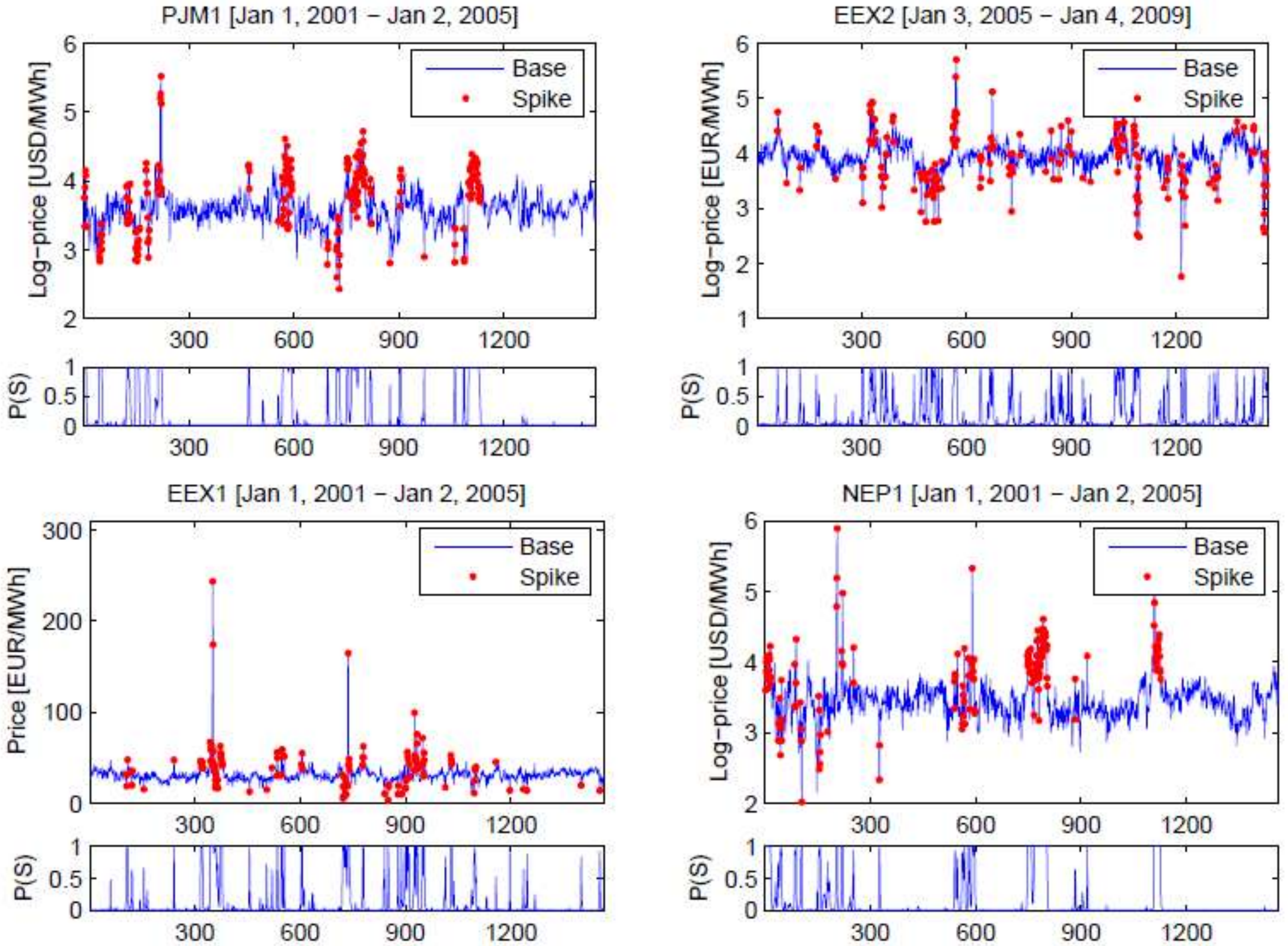


Figure 3: Sample calibration results for 2-regime models with Vasicek, i.e. AR(1), base regime dynamics and alternative spike regimes fitted to deseasonalized prices or log-prices from three major power markets. *Top left*: An independent spike (IS) model with normal spikes fitted to PJM log-prices. *Top right*: The Ethier and Mount (1998) model with AR(1) spike regime fitted to EEX log-prices. *Bottom left and right*: An IS model with lognormal spikes fitted to EEX prices and NEPOOL log-prices, respectively. The corresponding lower panels display the conditional probability $P(S) \equiv P(R_t = s|x_1, x_2, ..., x_T)$ of being in the spike regime. The prices or log-prices classified as spikes, i.e. with $P(S) > 0.5$, are additionally denoted by dots in the upper panels. For descriptions of the datasets see Section 2 and Figures 1-2.

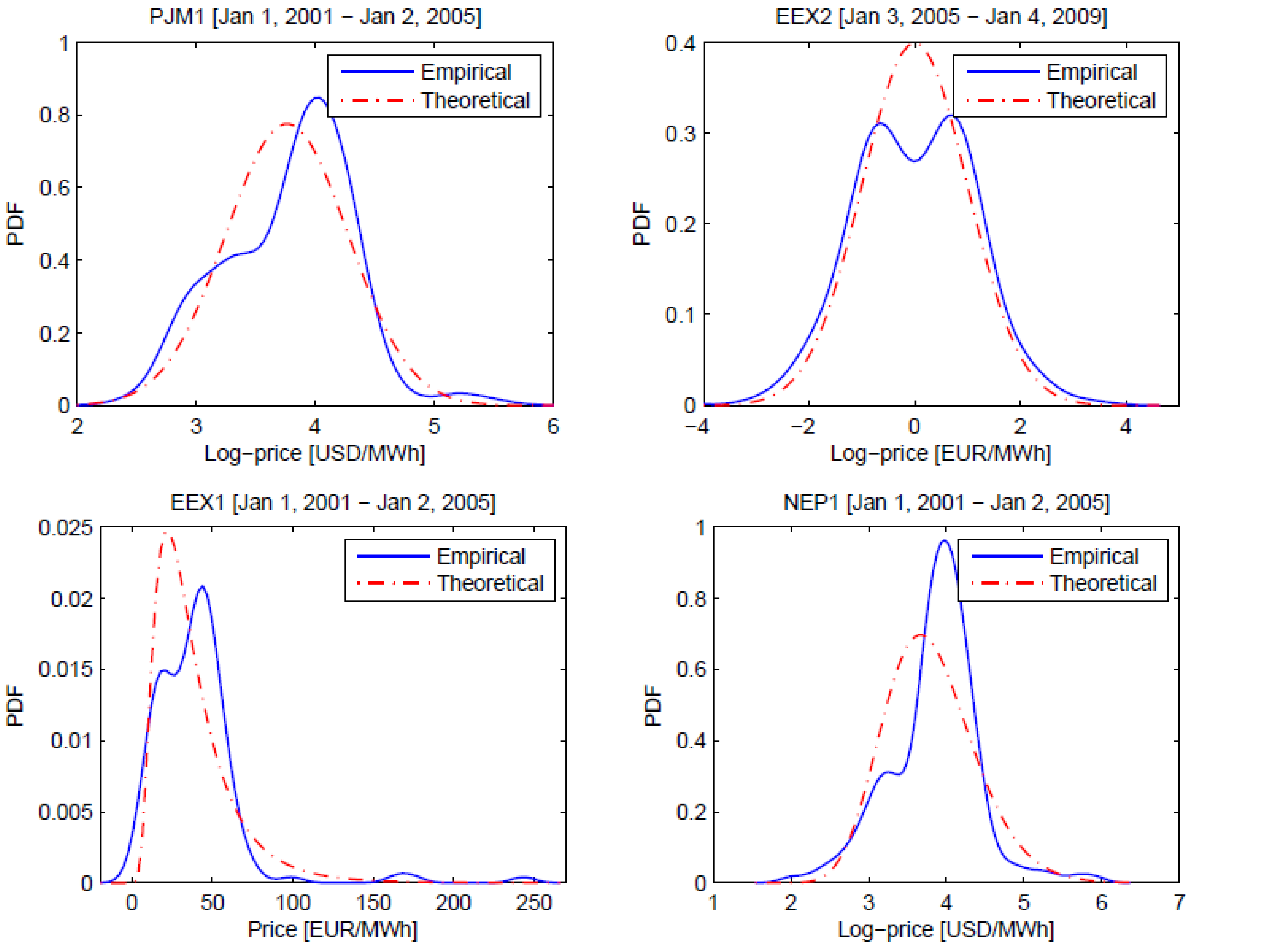


Figure 4: Comparison of empirical (sample) and theoretical (model implied) spike regime probability distribution functions in the first generation 2-regime models. The models and datasets are the same as in Figure 3. Note, that for the Ethier and Mount (1998) model the distributions of the noise in the AR(1) process driving the spike regime are plotted (*top right*).

Second generation MRS models for electricity (log–)spot prices

- Fundamental extensions to improve spike occurrence:
  - Mount et al. (2006) proposed a 2-regime model with
    - Two AR(1) regimes for log-prices and
    - Transition probabilities dependent on the reserve margin
  - Huisman (2008) extended the IS 2-regime model for log-prices
    - Considered temperature dependent transition probabilities
- Statistical refinements to improve goodness-of-fit:
  - Weron (2008) suggested to fit prices, not log-prices
  - Janczura and Weron (2009, 2010a, 2010b)
    - Introduced CIR-type dynamics for the base regime and
    - Median-shifted spike regime distributions
    - Advocated the IS 3-regime model

Goodness-of-fit

- Testing the goodness-of-fit for processes is not straightforward
- We can
  - Test the marginal distributions using an EDF-type test, like the Kolmogorov-Smirnov (K-S) test
- … but
  - The K-S test cannot be applied directly …
  - In the considered models neither the prices themselves nor their differences or returns are i.i.d.

Testing procedure #1: 'Equally weighted edf (ewedf)'

- Data is split into 2 subsets (3 for the 3-regime model)
  - **Spikes**, i.e. prices with probability P(R_t= 2) > 0.5
    - Price **drops**, i.e. prices with probability P(R_t= 3) > 0.5
  - The **base** regime, i.e. all remaining prices
- Discretization of the base regime dynamics:

$$dX_t = (\alpha - \beta X_t)dt + \sigma X_t^{\gamma} dW_t$$

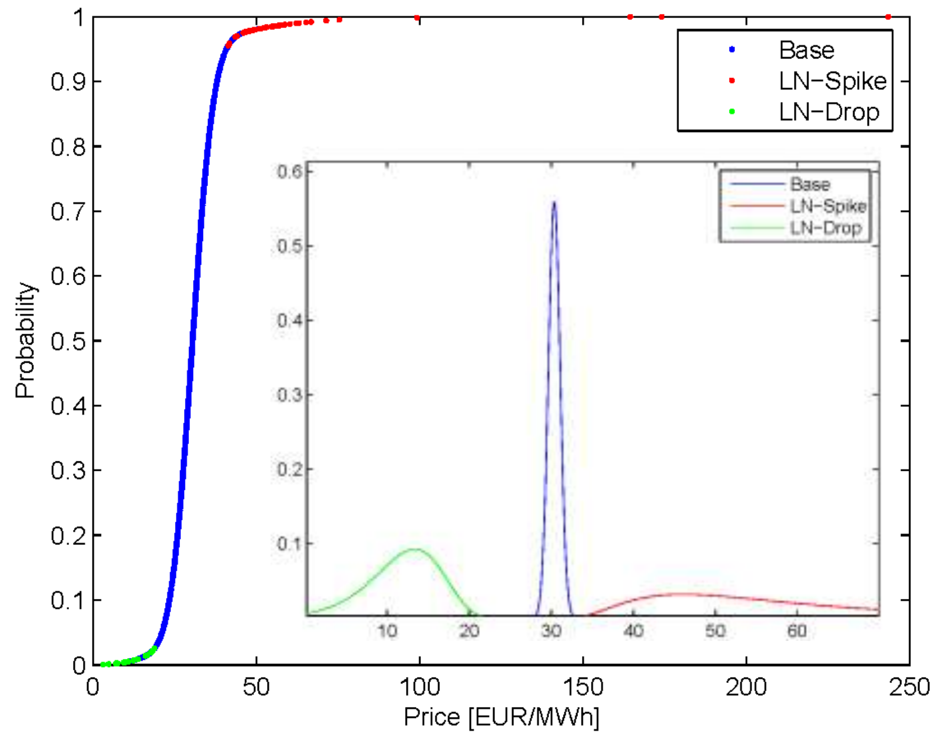
  - Where γ = 0 for the Vasicek model
- … for dt = 1 leads to:

$$\varepsilon_t = (X_{t+1} - (1-\beta)X_t - \alpha)/\sigma X_t^{\gamma}$$

  - Where ε_t's are i.i.d. Gaussian random variables

Testing procedure #1 cont.

- Applying the above transformation to base regime data we obtain 2 (or 3) i.i.d. samples:
  - f_S-distributed, e.g. lognormal or Pareto, for the spike regime
    - f_D-distributed, e.g. lognormal, for the drop regime
  - And Gaussian for the base regime



Testing procedure #1 cont.

- Combining these samples yields
  - A sample of independent variables with the distribution being a mixture of 2 (or 3) laws:
    - f_S, (f_D,) and Gaussian
- The probability that a given price X_t comes from
  - The spike distribution is equal to P(R_t = 2)
    - The price drop distribution is equal to P(R_t = 3)
  - The Gaussian law is equal to P(R_t = 1)
- We can perform the K-S test for
  - The subsets
  - And the whole sample

Testing procedure #2: 'Weighted edf (wedf)'

- Weighted empirical distribution function (edf)

$$F_n(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{I}_{\{X_i<t\}} \quad \Rightarrow \quad F_n(t) = \sum_{i=1}^{n} \frac{w_i \mathbb{I}_{\{X_i<t\}}}{\sum_{i=1}^{n} w_i} \quad \text{with} \quad w_i^j = P(R_i = j)$$

  - An unbiased and consistent estimator of F(t)
  - The statistics $D_n = \frac{\sum_{i=1}^{n} w_i}{\sqrt{\sum_{i=1}^{n} w_i^2}} \sup_{t\in\mathbb{R}} |F_n(t) - F(t)|$ converges (weakly) to the Kolmogorov-Smirnov distribution
  - For proofs see Janczura and Weron (2010b)

Comparison of #1 and #2

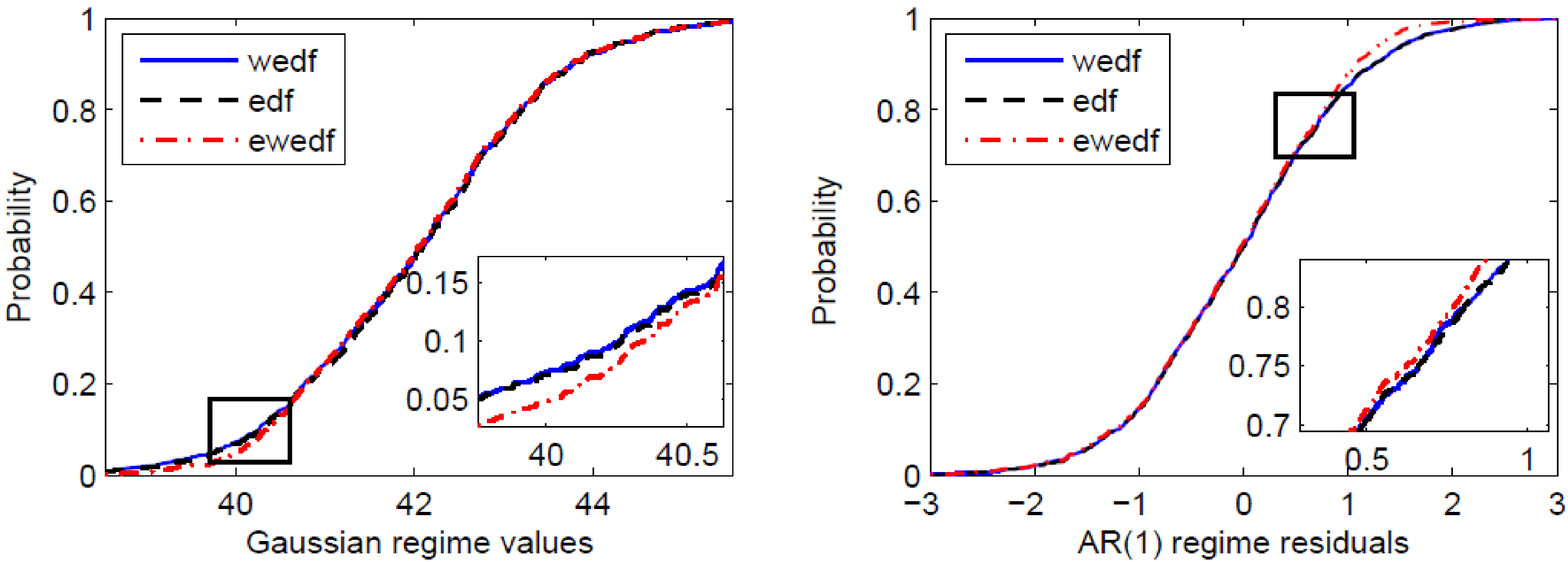


Figure 1: Comparison of the weighted empirical distribution function (wedf), the equally-weighted empirical distribution function (ewedf) and the standard empirical distribution function (edf) calculated for a sample trajectory of a MRS model with two independent regimes. Distribution functions of the i.i.d. Gaussian regime are given in the left panel, while of the residuals of the AR(1) regime in the right panel.

Shifted spike distributions

- Perhaps spike distributions should assign zero probability to prices below a certain quantile
- Let m = median(X_t)
  - Shifted log-normal (SLN), for $x > m$

$$\log(X_{t,2} - m) \sim N(\mu, \sigma^2)$$

  - Shifted Pareto (SP), for $x > \lambda \geq m$

$$X_{t,2} \sim F_P(x; \alpha, \lambda) = 1 - \left(\frac{\lambda}{x}\right)^{\alpha}$$

- Is the median cutoff optimal?
  - In general, no →

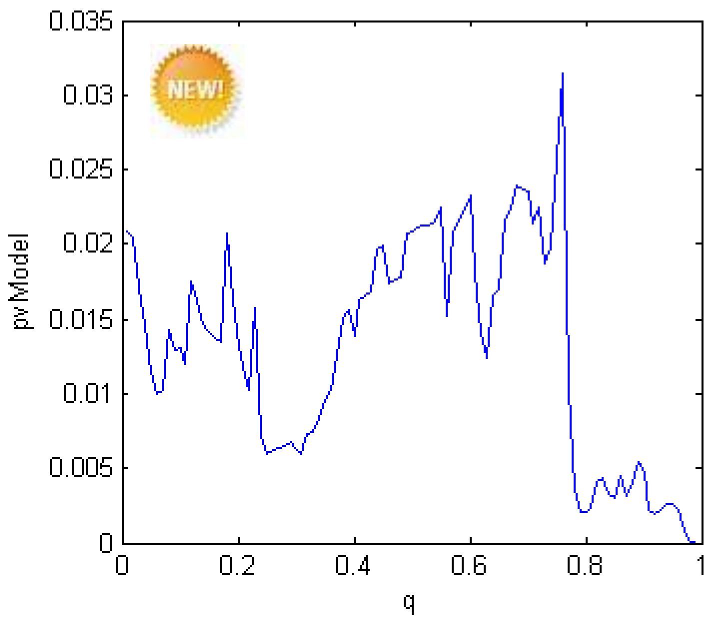


Table 1: Goodness-of-fit statistics for 2-regime models with Vasicek, see eqns. (4)-(5), base regime dynamics and median-shifted lognormal or Pareto spike distributions. Models for prices are summarized in columns 2-7, for log-prices in columns 8-13. p-values of 0.05 or more are emphasized in bold.

	Prices						Log-prices					
	Simulation			K-S test p-value			Simulation			K-S test p-value		
Data	IQR	IDR	LogL	Base	Spike	Model	IQR	IDR	LogL	Base	Spike	Model
					Shifted lognormal spikes							
EEX1	9%	11%	-4193.7	0.0012	**0.4061**	0.0032	30%	30%	403.0	0.0000	**0.7371**	0.0000
EEX2	13%	3%	-5066.9	0.0090	**0.4732**	0.0149	27%	16%	399.6	0.0000	**0.6313**	0.0000
PJM1	9%	-1%	-4385.9	0.0341	**0.4346**	**0.0530**	19%	8%	777.4	0.0007	**0.3219**	0.0012
PJM2	-3%	3%	-5012.1	**0.0887**	**0.9196**	**0.0893**	3%	6%	780.1	**0.0747**	**0.4147**	**0.0696**
NEP1	2%	2%	-4327.1	0.0247	**0.5093**	**0.0561**	9%	8%	610.4	0.0002	**0.8316**	0.0003
NEP2	0%	-2%	-4665.9	**0.0823**	**0.8416**	**0.1251**	8%	0%	1417.9	0.0088	**0.7430**	0.0170
					Shifted Pareto spikes							
EEX1	7%	9%	-4218.6	0.0000	0.0000	0.0000	27%	27%	436.9	0.0000	0.0123	0.0000
EEX2	10%	1%	-5101.8	0.0188	0.0008	0.0412	26%	16%	374.6	0.0000	0.0166	0.0000
PJM1	8%	-5%	-4447.2	**0.0500**	0.0000	**0.0500**	20%	6%	755.2	0.0007	0.0000	0.0012
PJM2	0%	1%	-5161.6	0.0041	0.0000	0.0007	6%	7%	744.9	0.0262	**0.3508**	0.0147
NEP1	-2	-6%	-4366.1	0.0300	0.0000	0.0300	8%	3%	546.7	0.0001	0.0000	0.0003
NEP2	13%	0%	-4703.1	0.0230	0.0000	0.0097	9%	0%	1409.7	0.0024	0.0000	0.0083

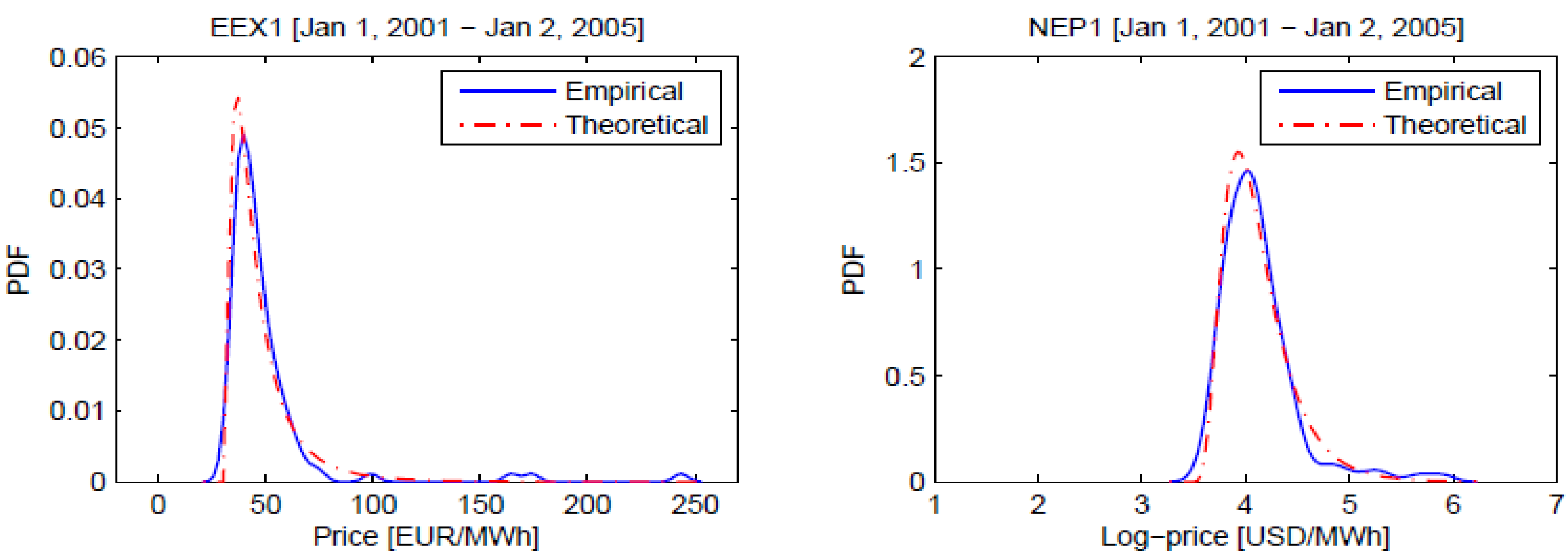
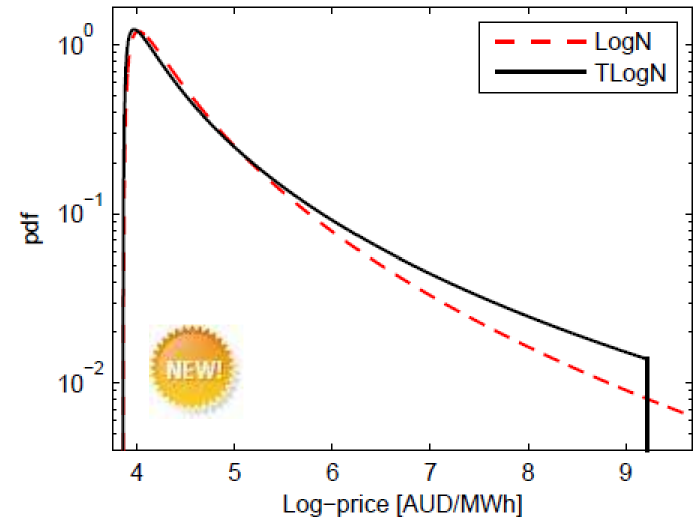


Figure 5: Comparison of empirical (sample) and theoretical (model implied) spike regime probability distribution functions in the 2-regime model with median-shifted lognormal spikes and Vasicek base regime dynamics. The fits are much better than for the models with non-shifted spike regime distributions, see Figure 4.

Price–capped spike distributions

- For extremely spiky markets (such as the Australian) they improve the fit
  - Bid-cap of $e^{9.21}$=10000 AUD



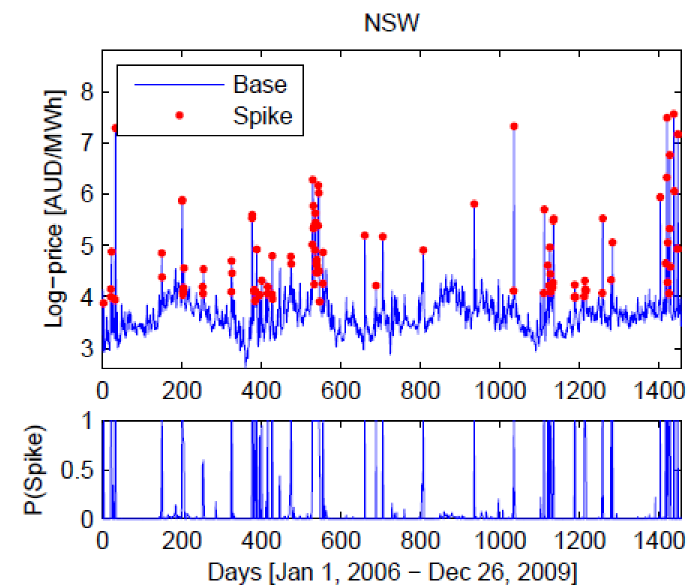
We consider the lognormal (LogN) distribution:

$$\log(X_t - m) \sim \mathrm{N}(\alpha_2, \sigma_2^2), \quad X_t > m, \tag{2}$$

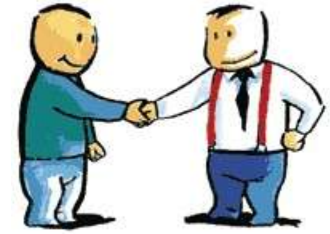
and the truncated lognormal (TLogN) distribution:

$$f(x) = \frac{C \exp\left(-\frac{(\log(x-m)-\alpha_2)^2}{2\sigma_2^2}\right)}{(x-m)\sigma_2\sqrt{2\pi}}, \quad x > m, \tag{3}$$

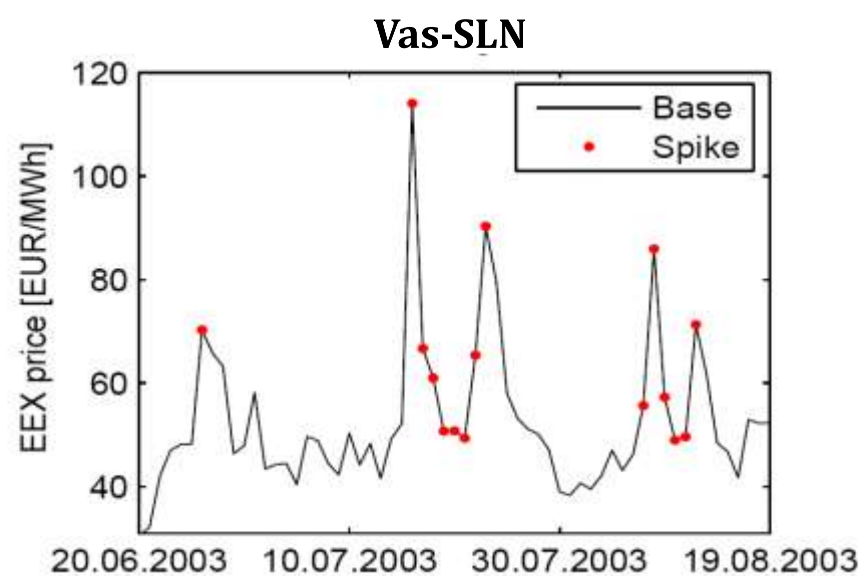
where $C = \Phi((\log(L) - \alpha_2)/\sigma_2)$ is a normalizing constant, Φ is the standard Gaussian cumulative distribution function (cdf) and L is the truncation level.



Conclusions



- When fitting MRS models with shifted spike distributions
  - Practically all spikes are identified correctly
  - There are no ‘sudden drops’ classified as spikes
- This suggests that the shifted distributions are more suitable for the spikes
  - But … there are clusters of ‘normal’ prices classified as spikes



Diffusion processes revisited

- Perhaps different dynamics should be used for the base regime
- NEW! Introduce heteroskedasticity ($\gamma \neq 0$)

$$dX_t = (\alpha - \beta X_t)dt + \sigma X_t^{\gamma} dW_t$$

  - For $\gamma=0$, Vasicek MRD process (Vasicek, 1977)
  - For $\gamma=0.5$, CIR square root process (Cox, Ingersoll, Ross, 1985)
  - For $\gamma=1$, BS process (Brennan, Schwartz, 1980)

Vasicek vs. heteroscedastic base regime dynamics

Table 2: Goodness-of-fit statistics for 2-regime models with heteroscedastic base regime dynamics and median-shifted lognormal spike distributions. Models for prices are summarized in columns 2-8, for log-prices in columns 9-15. p-values of 0.05 or more are emphasized in bold.

	Prices							Log-prices						
		Simulation			K-S test p-value				Simulation			K-S test p-value		
Data	γ	IQR	IDR	LogL	Base	Spike	Model	γ	IQR	IDR	LogL	Base	Spike	Model
Shifted lognormal spikes														
EEX1	-0.43	0%	0%	-4169.3	0.0022	**0.2365**	0.0050	-4.08	22%	26%	625.5	0.0000	**0.9865**	0.0000
EEX2	-0.32	10%	2%	-5041.7	0.0125	**0.2306**	0.0276	-3.69	22%	12%	551.8	0.0000	**0.5875**	0.0000
PJM1	0.10	5%	1%	-4356.4	**0.0853**	**0.5408**	**0.1607**	-1.02	17%	6%	793.1	0.0006	**0.1924**	0.0011
PJM2	0.16	1%	-1%	-4989.3	**0.5882**	**0.1802**	**0.5435**	-0.01	1%	2%	804.2	**0.0582**	**0.1843**	**0.0995**
NEP1	0.22	2%	0%	-4326.3	0.0317	**0.4754**	**0.0742**	-1.35	9%	12%	643.1	0.0003	**0.8524**	0.0003
NEP2	0.62	0%	0%	-4654.0	**0.0828**	**0.3566**	**0.0983**	-2.37	1%	-1%	1445.2	0.0368	**0.1724**	**0.0980**

Leverage effect ?!

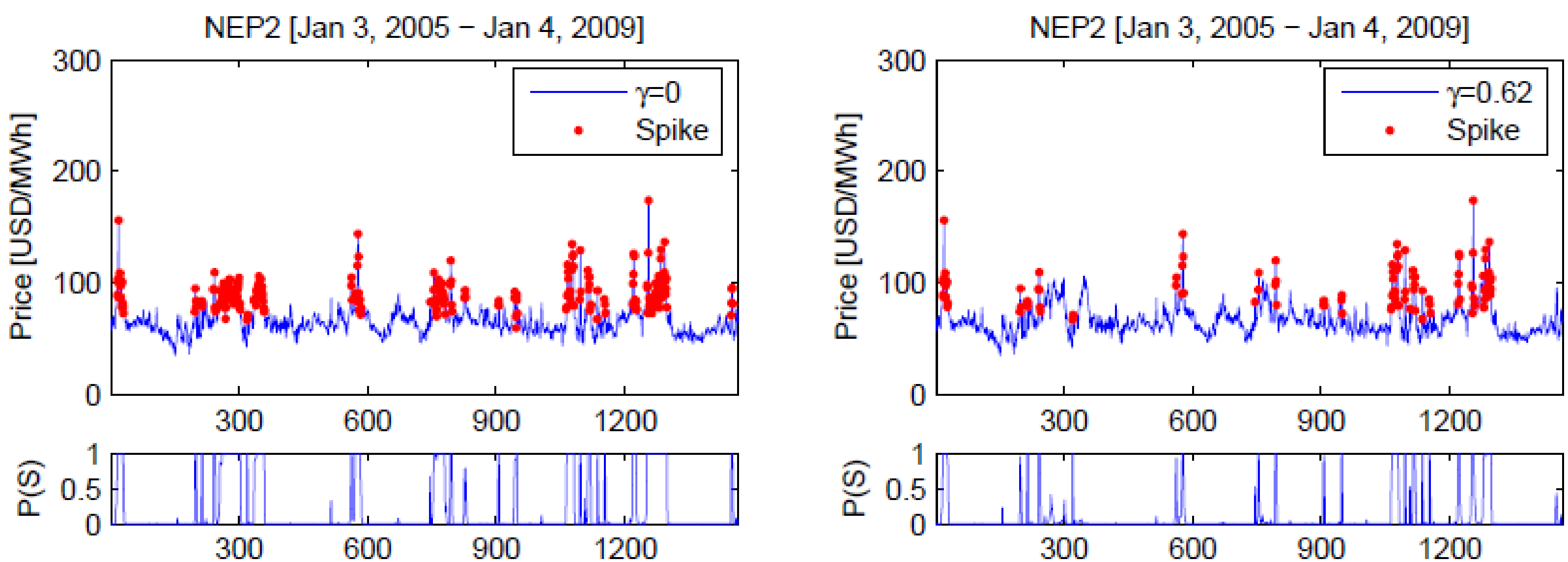
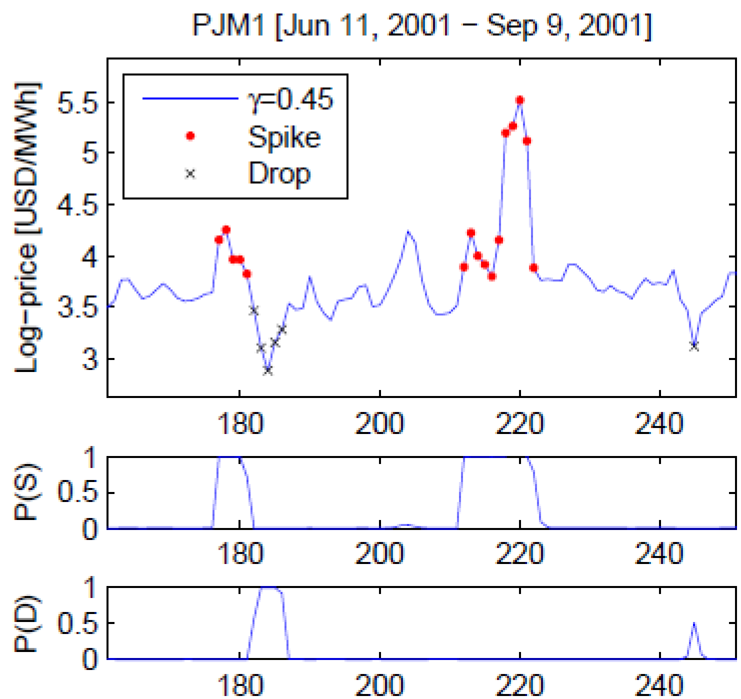


Figure 6: Sample calibration results for the 2-regime model with median-shifted lognormal spikes fitted to NEP2 prices. The difference between Vasicek (*left*) and heteroscedastic (*right*) base regime dynamics is clearly visible. Note, that due to the cutoff at the median, none of the price 'drops' are classified as spikes anymore.

3-regime models revisited

- Perhaps we need a 3rd regime to model the 'price drops'
- Introduce a 3rd 'drop' regime
  - Contrary to the Huisman & Mahieu (2003) model, the price **can stay** in the 'excited' regimes ('spike' and 'drop')
  - Use a 'mirror image' or 'reflected' shifted log-normal distribution



IS 3-regime models

Table 4: Goodness of fit statistics for the IS 3-regime models with heteroscedastic base regime dynamics and median-shifted lognormal spikes and drops. p-values of 0.05 or more are emphasized in bold.

Data	Simulation IQR	Simulation IDR	LogL	K-S test p-values Base	K-S test p-values Spike	K-S test p-values Drop	K-S test p-values Model
Prices							
EEX1	-1%	3%	-3798.2	**0.8371**	**0.0726**	**0.8576**	**0.5719**
EEX2	5%	-1%	-4848.5	**0.5510**	**0.2920**	**0.9196**	**0.3168**
PJM1	2%	0%	-4153.9	**0.3876**	**0.7052**	**0.7715**	**0.4072**
PJM2	2%	0%	-4723.2	**0.4824**	**0.3273**	0.0244	**0.4828**
NEP1	0%	0%	-4266.6	0.0404	**0.6121**	**0.9092**	**0.0609**
NEP2	5%	0%	-4610.3	**0.1359**	**0.7911**	**0.8771**	**0.1059**
Log-prices							
EEX1	-2%	5%	1181.2	**0.7297**	**0.3341**	**0.1971**	**0.5001**
EEX2	7%	1%	944.6	**0.2933**	**0.5090**	**0.5204**	**0.6517**
PJM1	8%	1%	1002.9	**0.3413**	**0.2726**	**0.9080**	**0.4640**
PJM2	1%	2%	1030.2	**0.2165**	**0.4604**	**0.2887**	**0.5258**
NEP1	1%	2%	853.0	**0.1084**	**0.8940**	**0.1948**	**0.1362**
NEP2	6%	0%	1573.7	**0.6858**	**0.7338**	**0.2334**	**0.8796**

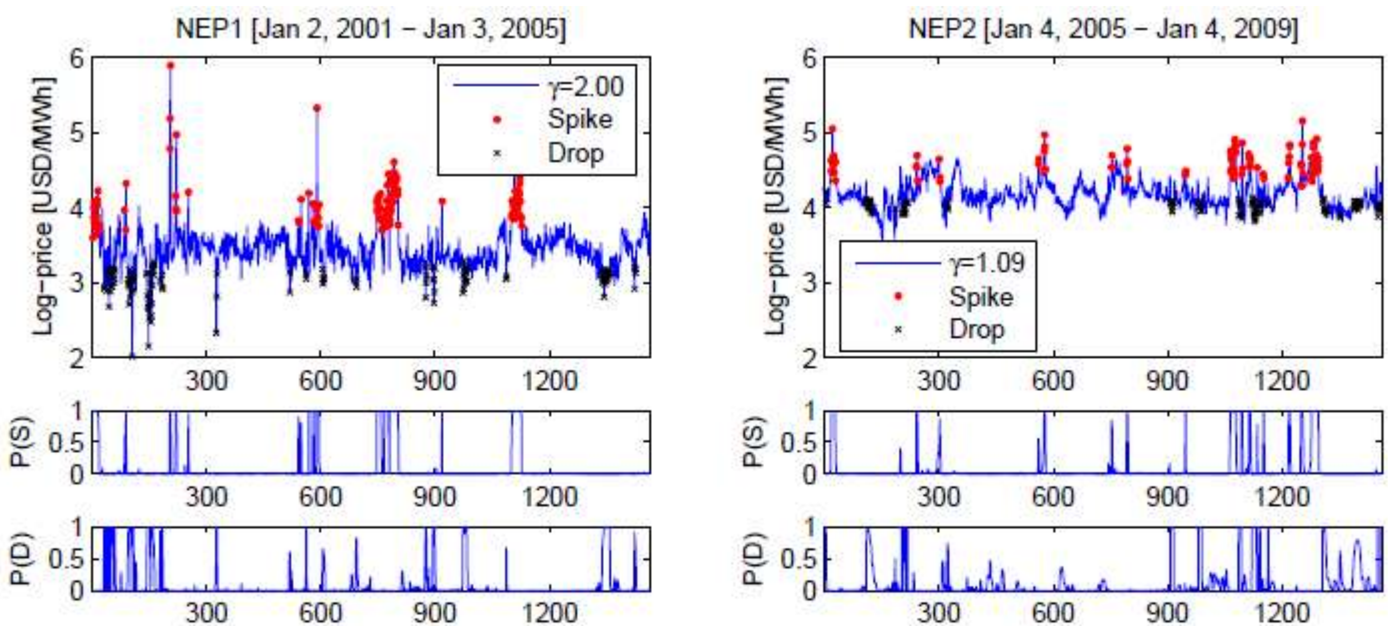


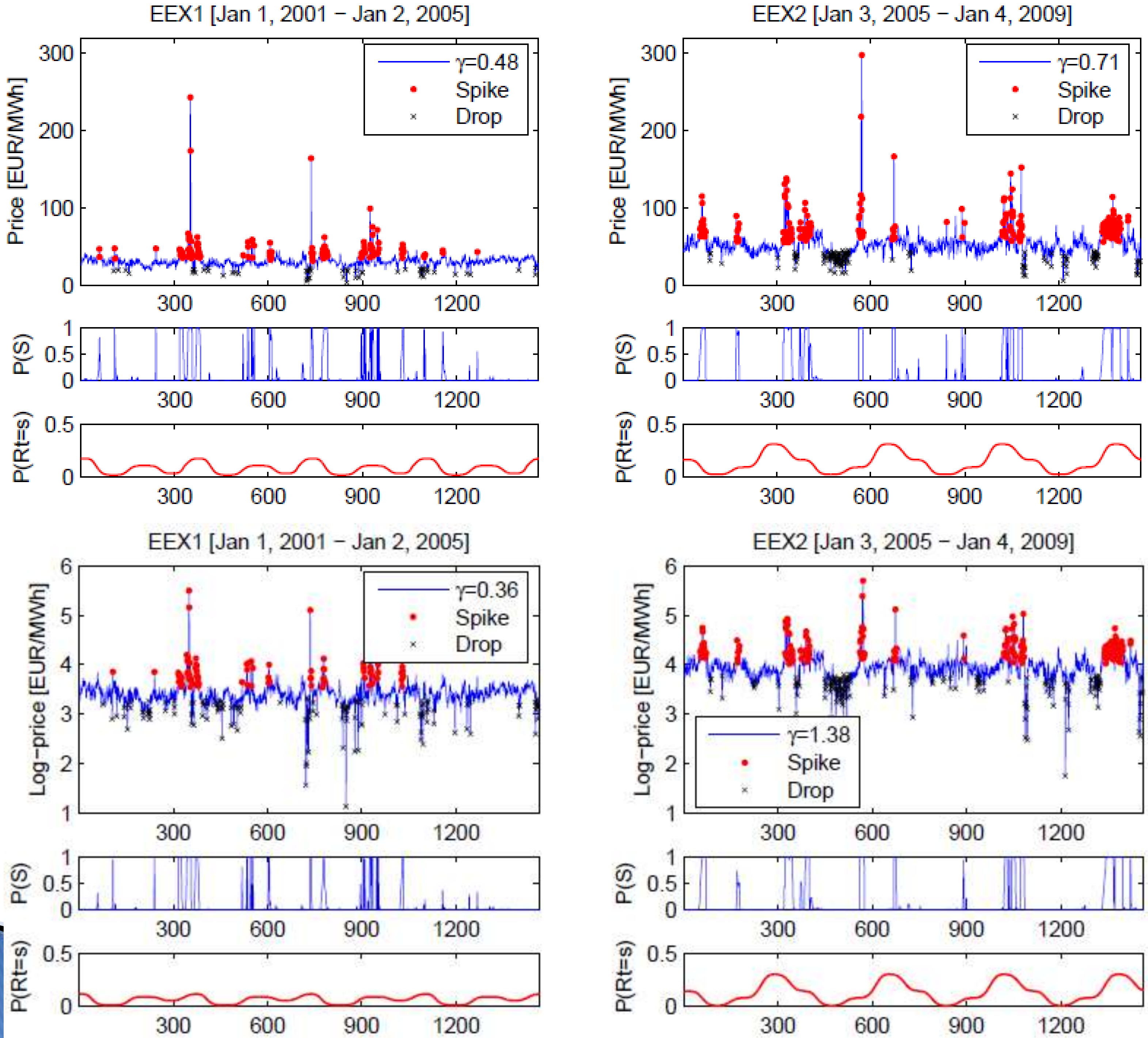
Figure 9: Calibration results for the IS 3-regime models with heteroscedastic base regime dynamics and median-shifted lognormal spikes and drops fitted to log-prices.

Inverse leverage effect !

Data	γ
Prices	
EEX1	0.6309
EEX2	0.3070
PJM1	0.6595
PJM2	0.1724
NEP1	0.5262
NEP2	0.0742
Log-prices	
EEX1	0.4102
EEX2	0.9115
PJM1	0.4481
PJM2	0.5057
NEP1	0.3068
NEP2	1.0923

IS 3-regime models with time-varying transition probabilities

- NEW! Admit a transition matrix with **time-varying (periodic) probabilities** $p_{ij}(t)$
- Calibrated in a two-step procedure in the last part of the E-step of the EM algorithm:
  - First, the probabilities are estimated independently for each season: Winter (XII-II), Spring (III-V), Summer (VI-VIII) and Autumn (IX-XI)
  - Then they are smoothed using a kernel density estimator with a Gaussian kernel
- This modification complicates gof testing
  - Only p-values for individual regimes are reported


EEX1 [Jan 1, 2001 − Jan 2, 2005]
EEX2 [Jan 3, 2005 − Jan 4, 2009]
Price [EUR/MWh]
γ=0.48
γ=0.71
Spike
Drop
P(S)
P(Rt=s)
Log−price [EUR/MWh]
γ=0.36
γ=1.38

Transition probabilities:

constant vs. time-varying

Data	Simulation IQR	Simulation IDR	LogL	K-S test p-values Base	K-S test p-values Spike	K-S test p-values Drop
			Prices			
EEX1	-0%	-2%	-3896.2	**0.9283**	**0.2444**	**0.5846**
EEX2	-4%	-1%	-4839.5	**0.1551**	**0.2536**	**0.8980**
PJM1	-5%	-2%	-4215.9	**0.1957**	**0.5965**	**0.3705**
PJM2	-4%	-4%	-4879.7	0.0424	**0.4312**	**0.1007**
NEP1	0%	0%	-4211.1	0.0134	**0.5258**	**0.7442**
NEP2	-3%	-2%	-4566.5	**0.2981**	**0.1140**	**0.4922**
			Log-prices			
EEX1	-1%	-3%	1132.7	**0.6898**	**0.8781**	0.0417
EEX2	-3%	0%	959.8	**0.3952**	**0.3190**	**0.2340**
PJM1	-2%	-3%	1018.7	**0.3139**	**0.1204**	**0.8435**
PJM2	-2%	-2%	904.2	**0.1272**	**0.0953**	**0.3402**
NEP1	0%	0%	863.6	**0.1881**	**0.8957**	**0.7062**
NEP2	-1%	-3%	1593.9	**0.7883**	**0.1881**	0.0409

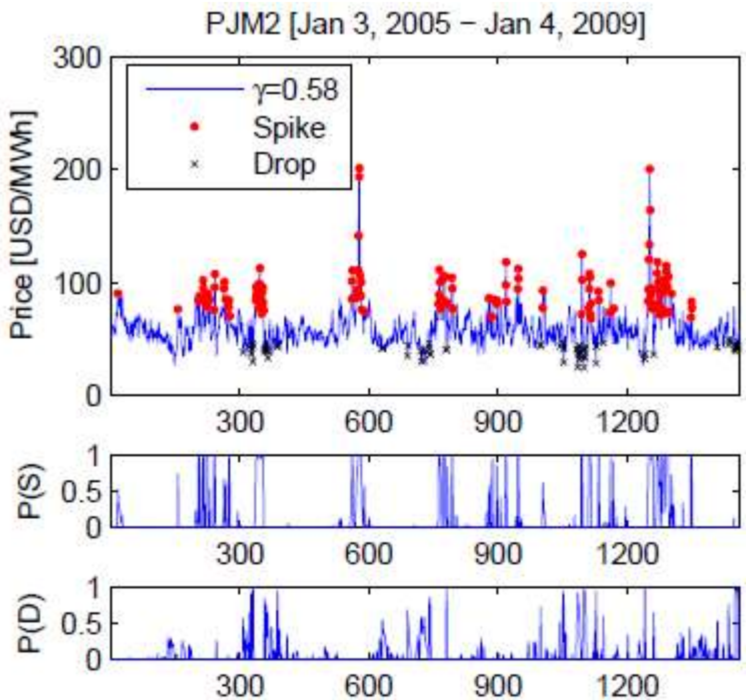
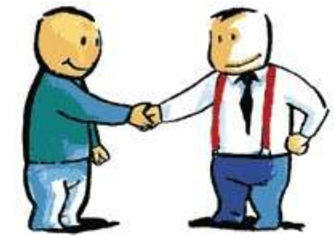


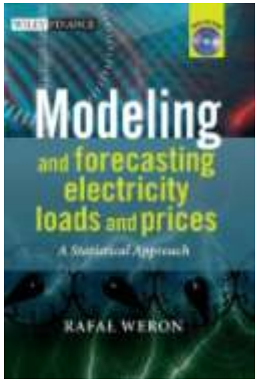
Figure 8: Comparison of calibration results for the IS 3-regime models with constant (*left*) and time-varying (*right*) transition probabilities. Note, the time-varying (periodic) intensity of spikes and price drops and the overall visually better fit of the latter model. The corresponding lower panels display the conditional probabilities $P(S) \equiv P(R_t = s|x_1, x_2, ..., x_T)$ and $P(D) \equiv P(R_t = d|x_1, x_2, ..., x_T)$ of being in the spike or drop regime, respectively.

Conclusions



- The **quest** for the **model** is not over
  - Improve the timing of spikes
- The **devil** is in **deseasonalization**
  - Preprocess data before fitting the seasonal components
  - Use fundamental data to better fit long term seasonality
- "Added value"
  - MRS models can be used to identify spikes in data
  - … but spike identification is dependent on specification of the models for the regimes

References



Based on:

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