A guide to robust modeling of electricity spot prices

Rafał Weron

Joint work with Joanna Janczura, Jakub Nowotarski, Jakub Tomczyk (Wrocław), Stefan Trück (Sydney) and Rodney Wolff (Brisbane)

Institute of Organization and Management
Wrocław University of Technology

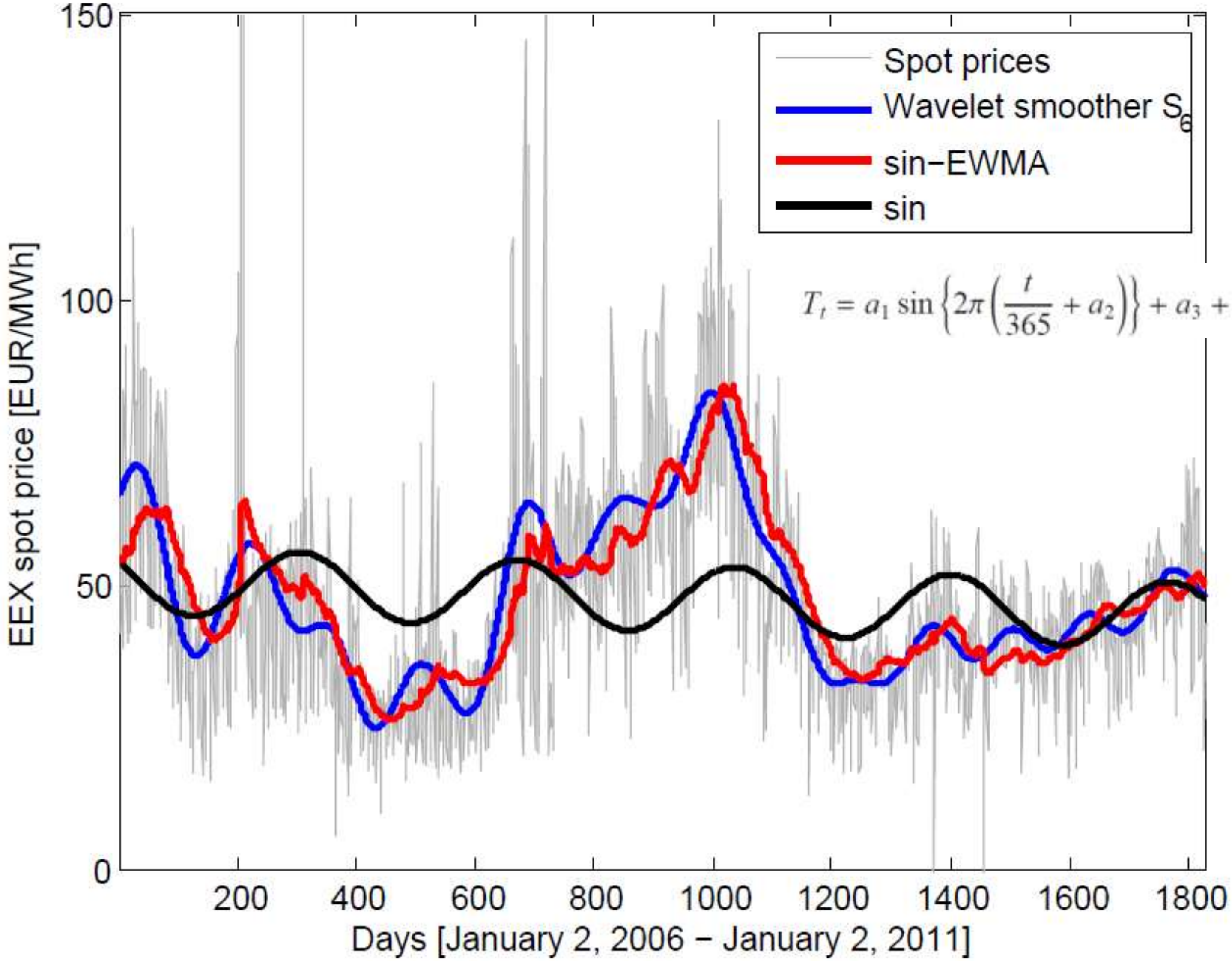
Introduction

- When building electricity spot price models we should address two questions:
  - How to estimate the trend-seasonal component?
  - How to forecast it?

- Why?

5 years of EEX spot prices:

3 fits of the long-term seasonal component (LTSC)



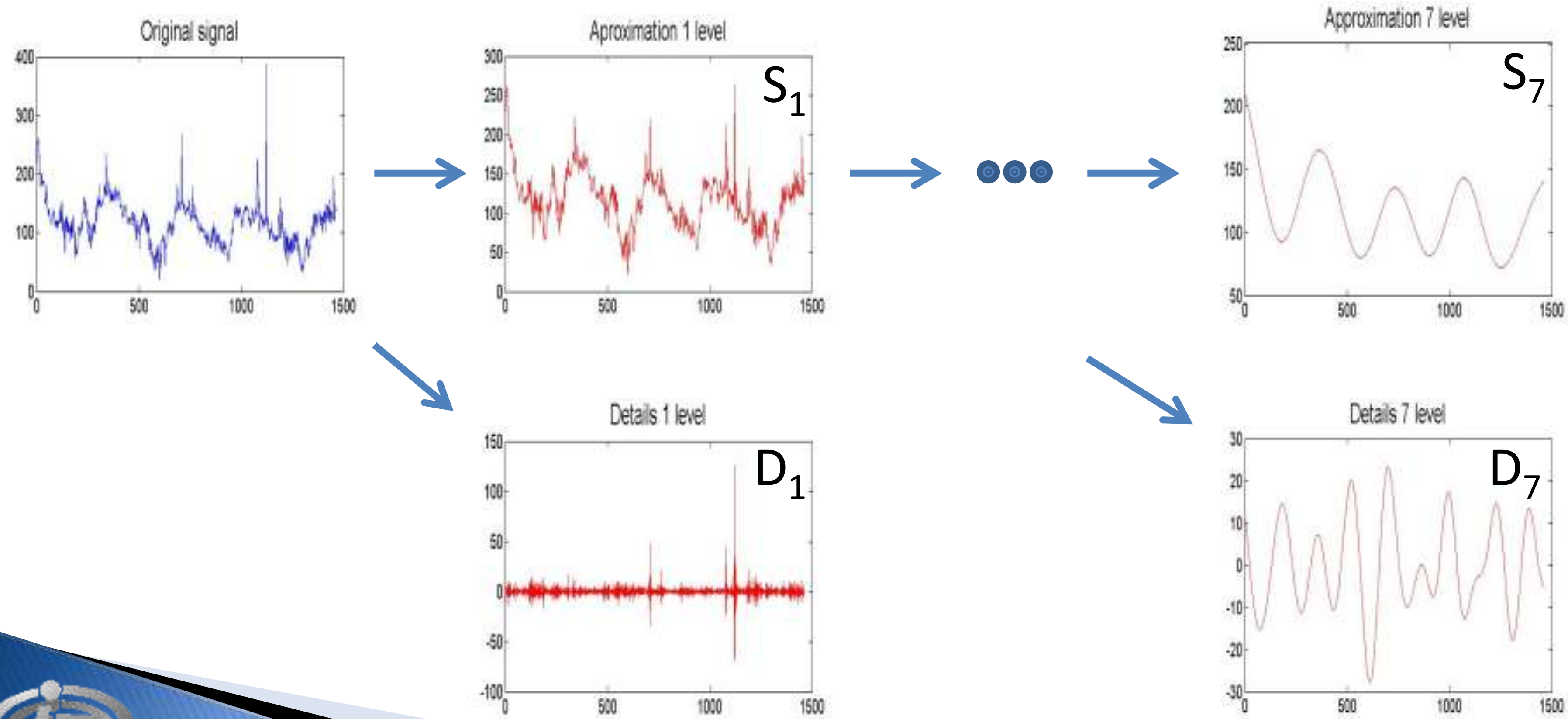
$$T_t = a_1 \sin\left\{2\pi\left(\frac{t}{365} + a_2\right)\right\} + a_3 + a_4 \mathrm{EWMA}_t^{0.975}$$

Modeling the LTSC

- Piecewise constant functions (or dummies) for the months
  - (Bhanot, 2000; Fanone et al., 2012; Fleten et al., 2011; Haldrup et al., 2010; Higgs and Worthington, 2008; Knittel and Roberts, 2005; Lucia and Schwartz, 2002)
- Sinusoidal functions or sums of sinusoidal functions of different frequencies
  - (Bierbrauer et al., 2007; Borovkova and Permana, 2006; Cartea and Figueroa, 2005; De Jong, 2006; Erlwein et al., 2010; Geman and Roncoroni, 2006; Keles et al., 2012; Lucia and Schwartz, 2002; Pilipovic, 1998; Seifert and Uhrig-Homburg, 2007; Weron, 2008)
- Wavelet smoothers
  - (Janczura and Weron, 2010, 2012; Stevenson, 2001; Stevenson et al., 2006; Weron, 2006, 2009; Weron et al., 2004a,b)

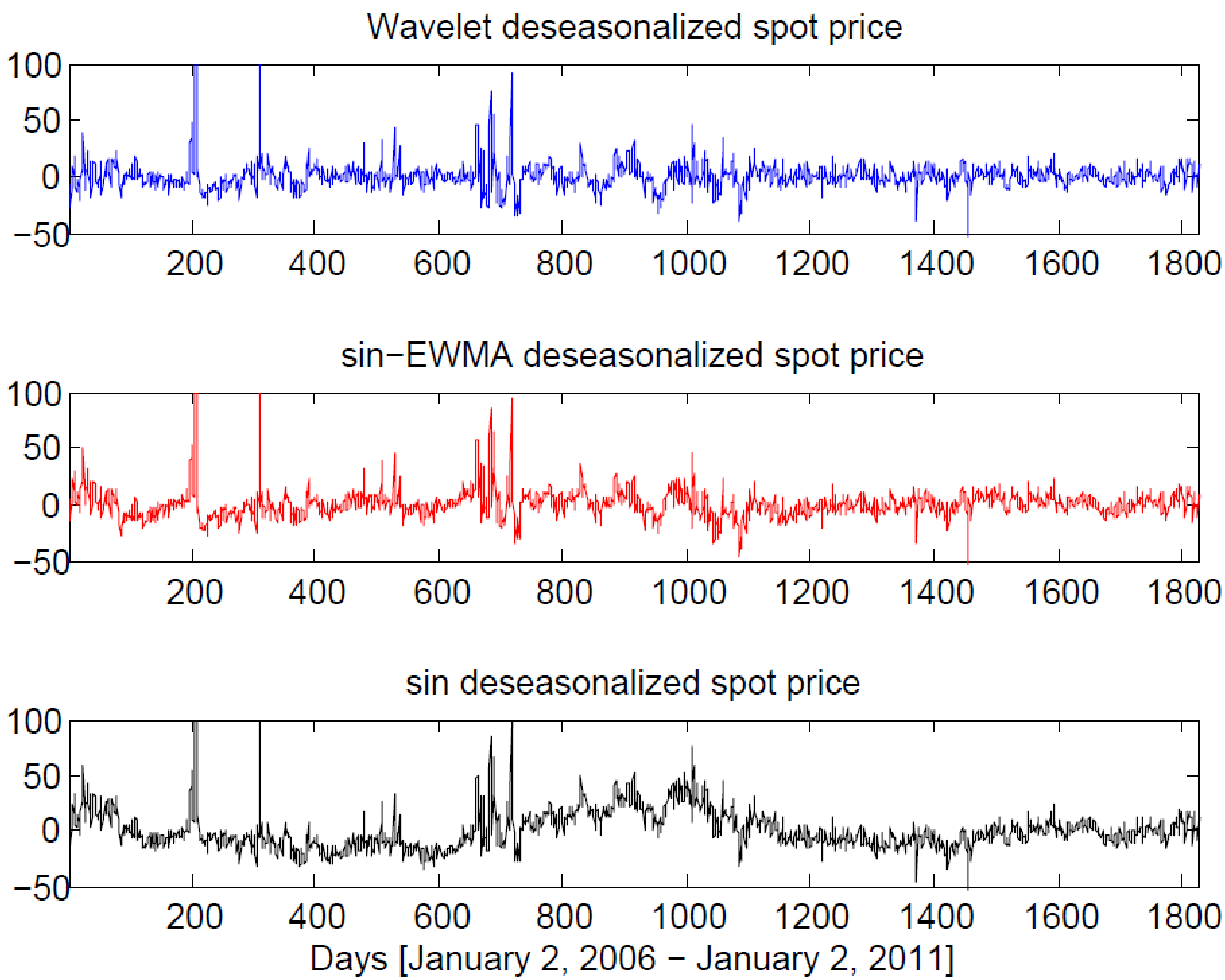
How do wavelets work?

- Decompose: $P(t) = S_J + D_J + D_{J-1} + \ldots + D_1$, with $2^J <$ #obs
  - S_7 roughly corresponds to `seasonal' (128 day) smoothing



3 deseasonalized EEX spot prices:

LTSC + STSC (daily + holiday dummies)



MRJD fits to deseasonalized prices

- Mean-reverting jump diffusion (MRJD):

$$dX = (\boldsymbol{\alpha} - \boldsymbol{\beta} X)dt + \boldsymbol{\sigma} dB + N(\boldsymbol{\mu}, \boldsymbol{\gamma})dP(\boldsymbol{\lambda})$$

	alpha	beta	sigma	mu	gamma	lambda
wavelet S_6	-0.366	0.405	6.442	8.386	40.80	0.0446
sin-EWMA	-0.350	0.321	6.436	7.242	39.87	0.0488
sin	-0.131	0.126	6.836	2.735	43.79	0.0499

MRS fits to deseasonalized prices

- Markov-regime switching (MRS):

$$X_t = \begin{cases} X_{t,1} & \text{if } R_t = 1, \quad X_{t,1} = \alpha_1 + (1-\beta_1)X_{t-1,1} + \sigma_1\epsilon_t, \\ X_{t,2} & \text{if } R_t = 2, \quad \log(X_{t,2} - X(q_2)) \sim \mathrm{N}(\mu_2, \sigma_2^2), \quad X_{t,2} > X(q_2). \\ X_{t,3} & \text{if } R_t = 3. \quad \log(-X_{t,3} + X(q_3)) \sim \mathrm{N}(\mu_3, \sigma_3^2), \quad X_{t,3} < X(q_3). \end{cases}$$

  - Transition matrices $\{p_{ij}\}$:

	wavelet S_6				sin-EWMA				sin			
	p_{i1}	p_{i2}	p_{i3}	**$P(R_t=i)$**	p_{i1}	p_{i2}	p_{i3}	**$P(R_t=i)$**	p_{i1}	p_{i2}	p_{i3}	**$P(R_t=i)$**
p_{1j}	0.97	0.02	0.01	**0.90**	0.97	0.02	0.01	**0.91**	0.98	0.02	0.01	**0.81**
p_{2j}	0.40	0.60	0.00	**0.05**	0.29	0.71	0.00	**0.06**	0.11	0.89	0.00	**0.11**
p_{3j}	0.17	0.05	0.79	**0.05**	0.22	0.02	0.76	**0.04**	0.09	0.00	0.91	**0.07**

Agenda

- Introduction
- **Case study I: Spikes and seasonality**
  - J.Janczura, S.Trück, R.Weron, R.Wolff (2012) *Identifying spikes and seasonal components in electricity spot price data: A guide to robust modeling*, RePEc/SSRN Paper
- Case study II: Forecasting LTSC

Seasonality and spikes vs. model estimation

- Spot price P_t (S_t) is typically modeled as a sum (or a product) of
  - a ‘deterministic’ (seasonal) component Λ_t and
  - a purely stochastic component X_t
- Before estimating the stochastic model for X_t, usually a seasonal model for Λ_t is calibrated
- Preprocessing - detecting & replacing price spikes – **might** improve the estimation of Λ_t … and of X_t

Research questions

- **How to define a price spike?**
- What is the influence of the chosen technique on the estimation of Λ_t and X_t
- How can we evaluate which of the suggested techniques works best?

Detecting price spikes

- **F**ixed **P**rice **T**hresholds
  → **FPT (±40 des. prices, ±0.5 des. log-prices)**
  (Boogert & Dupont, 2008; Lapuerta & Moselle, 2001)
- **V**ariable **P**rice **T**hresholds, e.g.
  (i) upper (and lower) q% of prices → **VPT1 (q=2.5%), VPT2 (q=10%)**
  (Trück et al., 2007)
  (ii) prices > 3 SD ('**R**ecursive **F**ilter on **P**rices') → **RFP**
  (Janczura et al., 2012)
- Fixed price change thresholds, e.g. returns > 30%
  (Bierbrauer et al., 2004)
- Variable price change thresholds ('**R**ecursive **F**ilter on price **D**ifferences'), e.g. > 3 SD → **RFD**
  (Cartea & Figueroa, 2005; Clewlow & Strickland, 2000; Weron, 2008)

Detecting price spikes cont.

- Wavelet filtering
  (Janczura & Weron, 2010; Stevenson, 2001; Stevenson et al., 2006; Weron, 2006)
- Thresholds implied by Gaussian 90% prediction intervals
  (Borovkova & Permana, 2006)
- Thresholds yielding the best model in terms of matching kurtosis
  (Geman & Roncoroni, 2006)
- '**R**ecursive seasonal **M**odel' (Trück et al., 2008) → **RM** (stop when the reduction in MSE of the LTSC is <1%)
- M**RS** model **C**lassification → **RSC**
  (Janczura & Weron, 2010 ; Janczura et al., 2012)

Replacing the spikes (outliers)

- Replace observed outliers by
  - The chosen threshold (Shahidehpour et al., 2002)
  - The mean of the two neighboring prices (Weron, 2008)
  - One of the neighboring prices (Geman & Roncoroni, 2006)
  - ‘Similar day’ values, e.g. the median of all prices having the same weekday and month (Bierbrauer et al., 2007)
  - **The mean of the deseasonalized prices (i.e., the LTSC)** (Janczura et al., 2012)
  - Do not replace them at all → delete them

The data

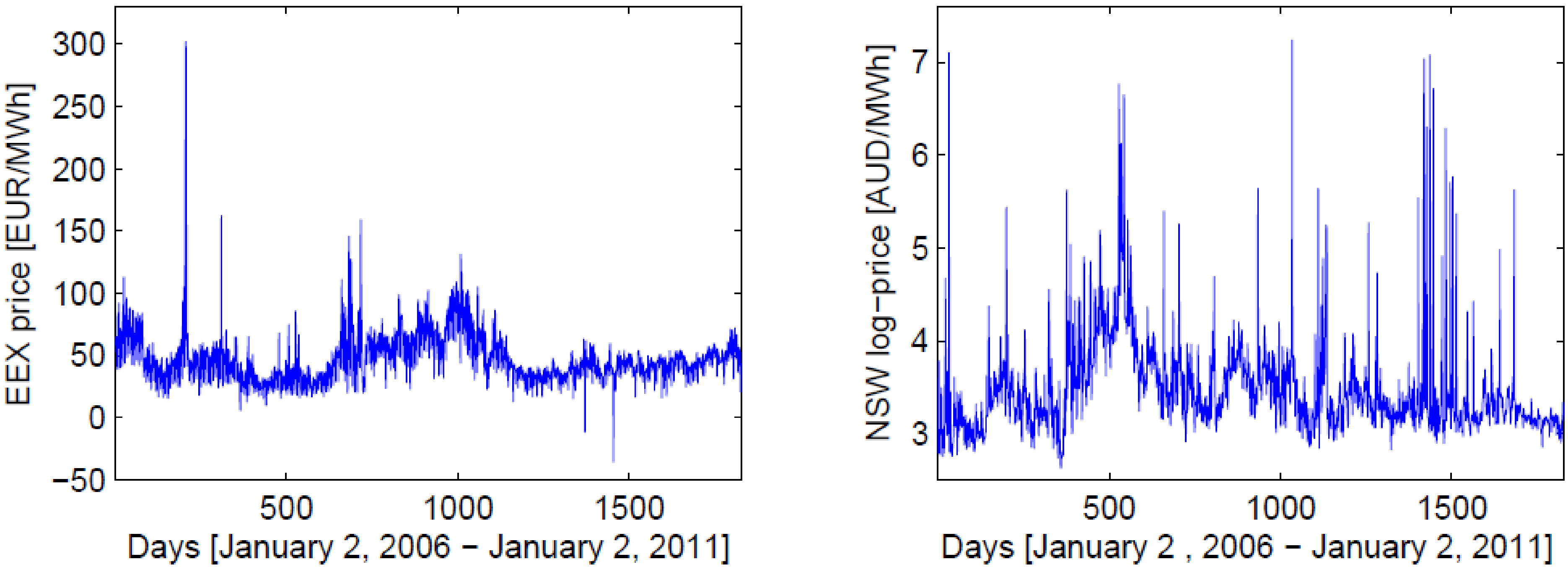
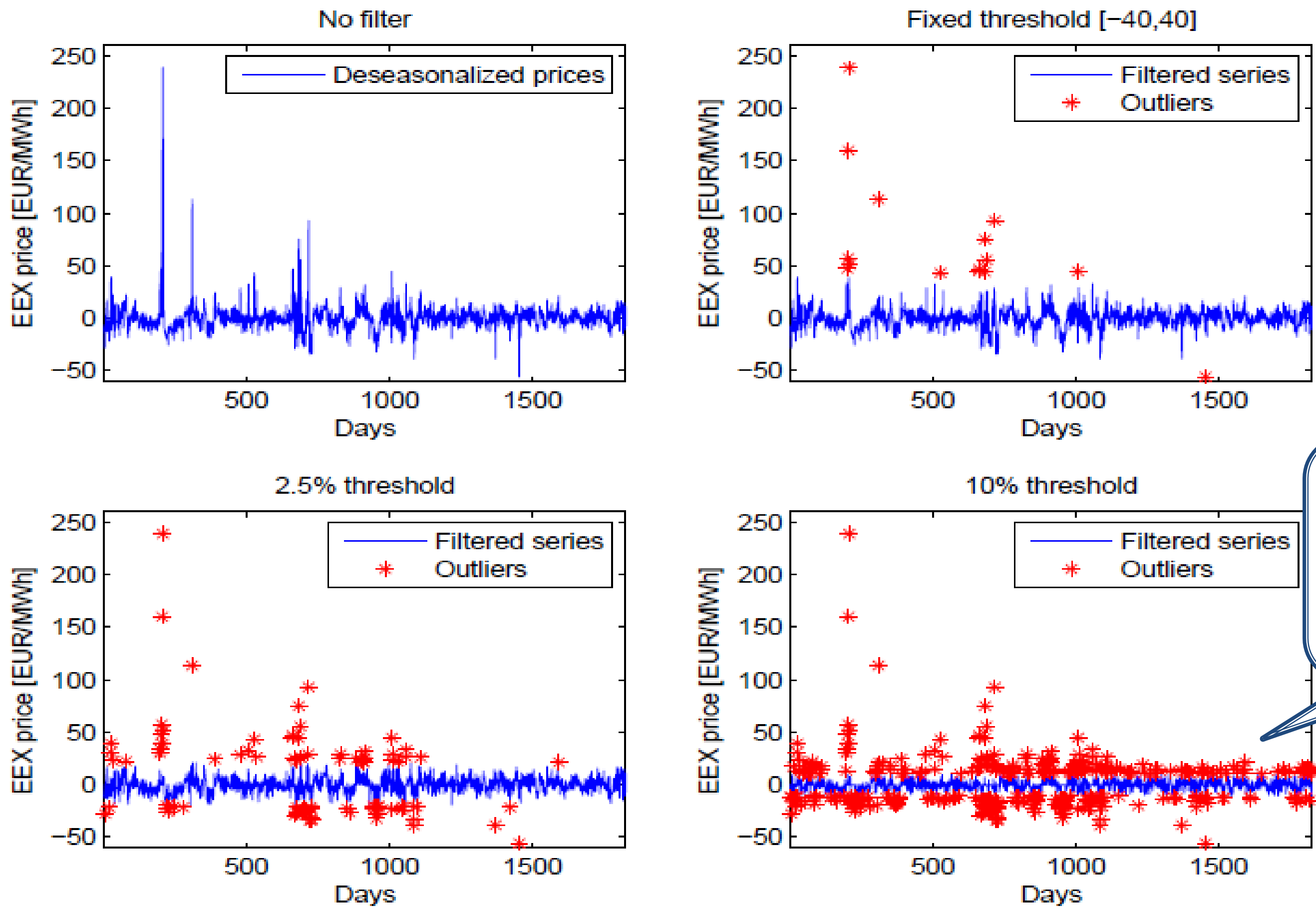


Figure 2: Mean daily electricity spot prices from the European Energy Exchange (EEX, Germany; *left*) and the New South Wales Electricity Market (NSW, Australia; *right*). Note that, in the right panel, the log-prices (and not the prices themselves) are plotted.

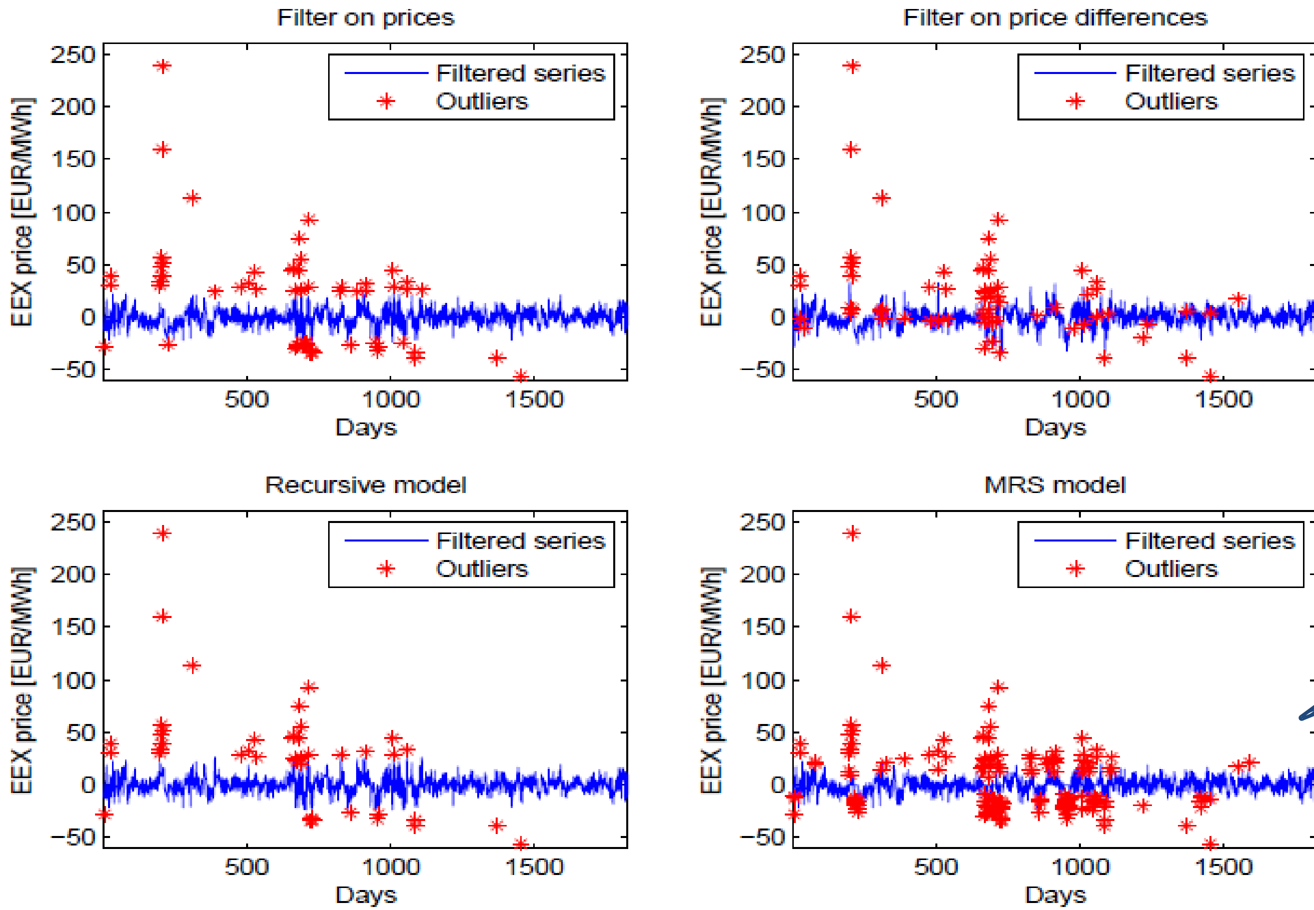
Outlier filtering (EEX)



VPT2 yields the highest number of outliers (20%)

Figure 3: Price observations for the EEX market that are identified as price spikes or price drops based on different methods for outlier detection (ORG, FPT, VPT1 and VPT2) *(left to right, top to bottom)* and wavelet LTSC.

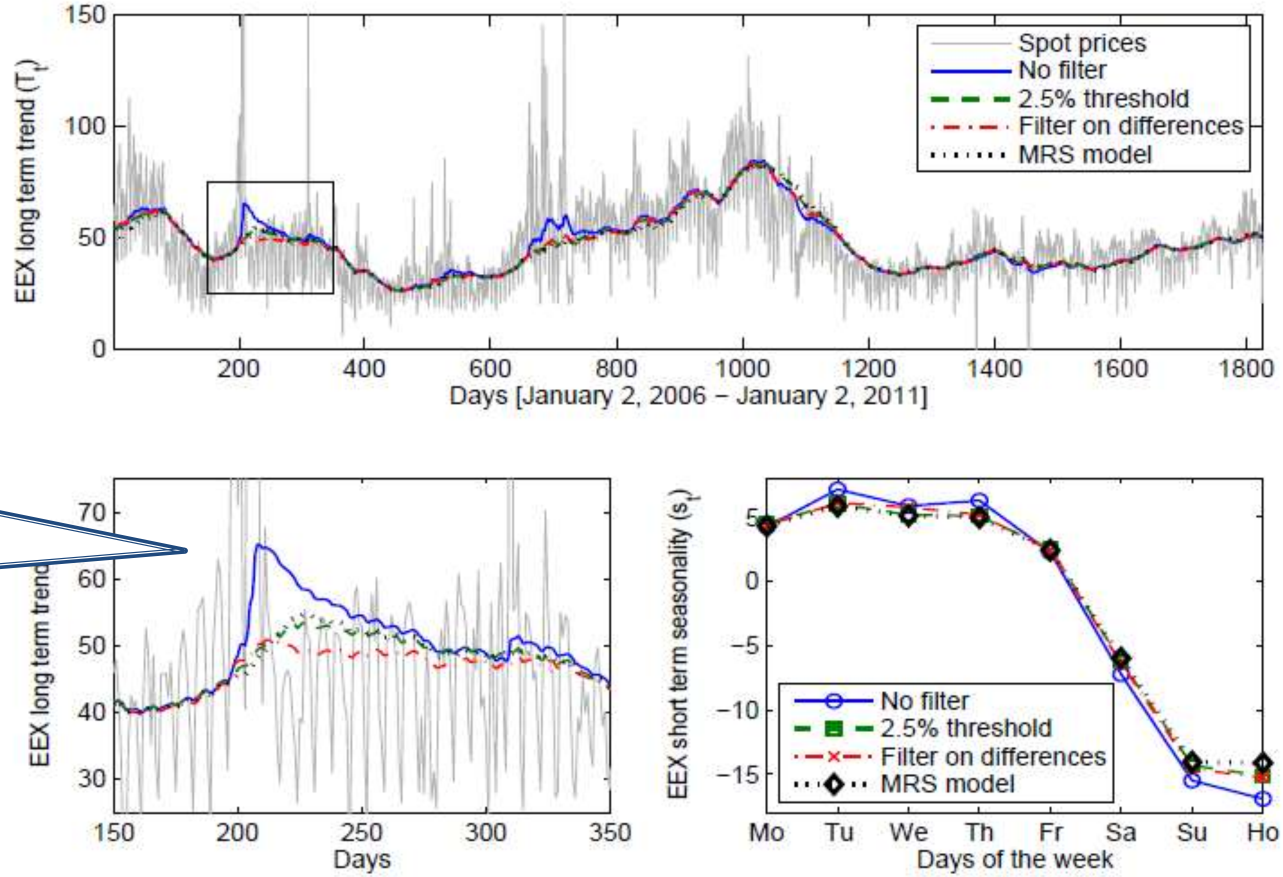
Outlier filtering (EEX) cont.



RSC yields 2^{nd} highest number of outliers

Figure 4: Price observations for the EEX market that are identified as price spikes or price drops based on different methods for outlier detection (RFP, RFD, RM and RSC) *(left to right, top to bottom)* and wavelet LTSC.

Seasonal pattern estimation: sin-EWMA



sin-EWMA estimated LTSC reacts with a delay

Figure 5: Price observations, estimated long-term trend T_t and short-term seasonality s_t for the EEX market. The *upper panel* shows the price series and the sin-EWMA LTSC. The lower panels provide a zoom into the LTSC for the period June to December 2006 and the estimated STSC for the filters.

Seasonal pattern estimation: Wavelet S_6

Wavelet S_6 approximation provides a much smoother LTSC than sin-EWMA

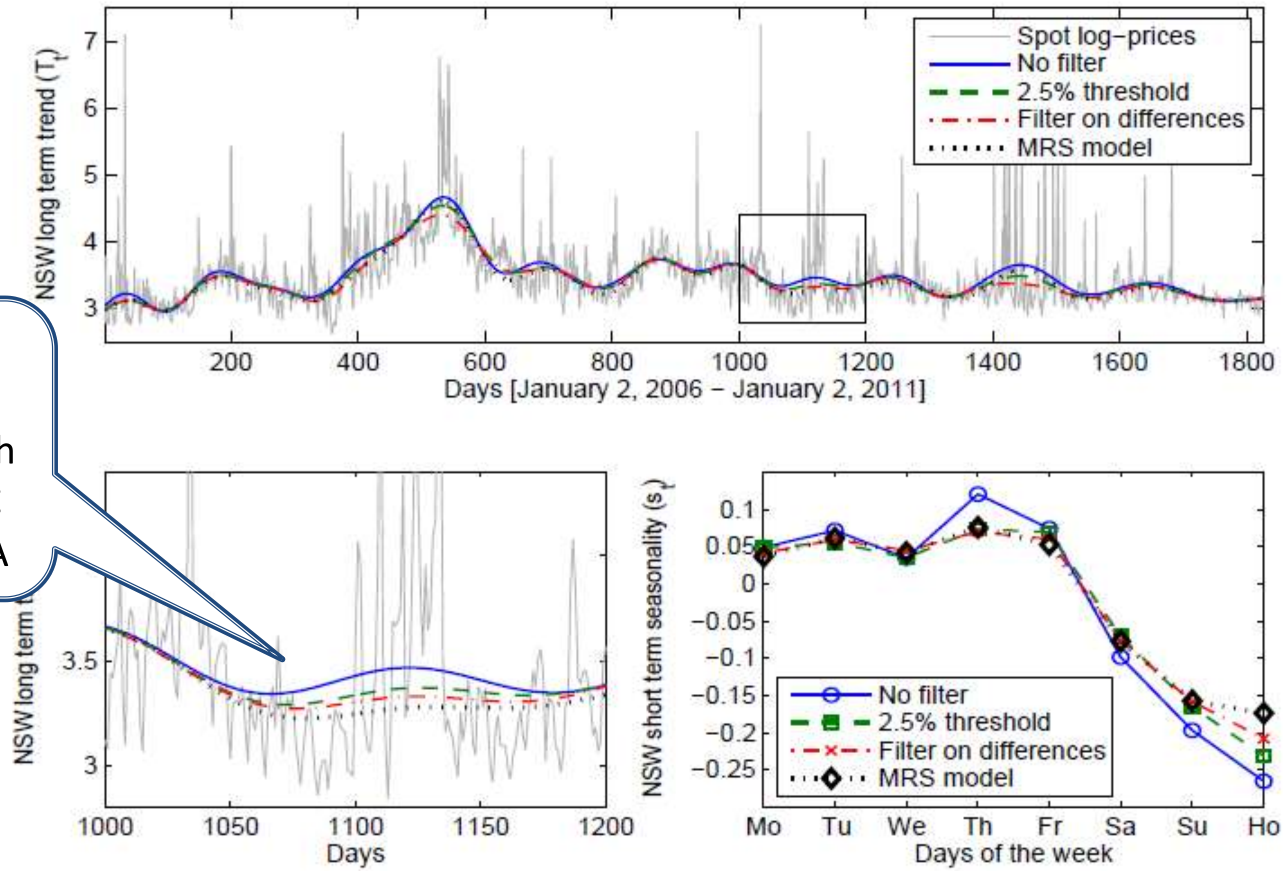


Figure 6: Price observations, estimated long-term trend T_t and short-term seasonality s_t for the NSW market. The *upper panel* shows the price series and the wavelet LTSC. The lower panels provide a zoom into the LTSC for the period October 2008 to April 2009 and the estimated STSC for the filters.

Monte Carlo simulation

- Apply different outlier detection methods (filters) to simulated trajectories
  - Then compare the resulting estimated parameters with the \`true' values
- The \`true'/\`realistic' LTSC and STSC
  - Estimated from the electricity spot prices using S_6 wavelet or sin-EWMA in combination with VPT1
- Stochastic trajectories (N=1000) are simulated from a MRS model with known parameters

Evaluation of the results

- Results are compared
  - for 2 markets (EEX, NSW) and
  - 2 estimation LTSC techniques (wavelet, sin-EWMA)
  - … across 8 filtering techniques
- We apply a *multiple comparison procedure* using Tukey's honestly significant difference criterion (Hochberg and Tamhane, 1987)
  - indicating for each of the methods which of the other filters performs significantly worse/better wrt MSE

Results for seasonal pattern estimation (wavelet) for EEX

Table 1: Mean Squared Error (MSE) for estimation of long-term (T_t), short-term (s_t) and overall (f_t) seasonal pattern for the conducted simulation study of the EEX markets with wavelet LTSC estimation.

	EEX (Wavelet)					
Measure	MSE (T_t)	Worse / Better	MSE (s_t)	Worse / Better	MSE (f_t)	Worse / Better
(1) ORG	7.7268	– / All	0.4297	– / All	8.1589	– / All
(2) FPT	5.7674	$\{1,4\}$ / $\{3,5,6,7,8\}$	0.2396	$\{1,4\}$ / $\{3,5,6,7,8\}$	5.9804	$\{1,4\}$ / $\{3,5,6,7,8\}$
(3) VPT1	**4.6753**	$\{1,2,4,6\}$ / –	0.1989	$\{1,2,4\}$ / $\{8\}$	**4.8485**	$\{1,2,4,6\}$ / –
(4) VPT2	6.3848	$\{1\}$ / $\{2,3,5,6,7,8\}$	0.2889	$\{1\}$ / $\{2,3,5,6,7,8\}$	6.6337	$\{1\}$ / $\{2,3,5,6,7,8\}$
(5) RFP	5.6470	$\{1,2,4,6\}$ / –	0.2082	$\{1,2,4\}$ / $\{8\}$	5.8160	$\{1,2,4,6\}$ / –
(6) RFD	5.2237	$\{1,2,4\}$ / $\{3,5,7,8\}$	0.1985	$\{1,2,4\}$ / $\{8\}$	5.3864	$\{1,2,4\}$ / $\{3,5,7,8\}$
(7) RM	4.8250	$\{1,2,4,6\}$ / –	0.2020	$\{1,2,4\}$ / $\{8\}$	4.9899	$\{1,2,4,6\}$ / –
(8) RSC	4.9798	$\{1,2,4,6\}$ / –	**0.1778**	$\{1,2,3,4,5,6,7\}$ / –	5.1229	$\{1,2,4,6\}$ / –

Results for seasonal pattern estimation (wavelet) for NSW

Table 2: Mean Squared Error (MSE) for estimation of long-term (T_t), short-term (s_t) and overall (f_t) seasonal pattern for the conducted simulation study of the NSW markets with wavelet LTSC estimation.

	NSW (Wavelet)					
Measure	MSE (T_t)	Worse / Better	MSE (s_t)	Worse / Better	MSE (f_t)	Worse / Better
(1) ORG	0.0106	– / {3,4,5,6,7,8}	0.0066	– / {3,4,5,6,7,8}	0.0113	– / {3,4,5,6,7,8}
(2) FPT	0.0105	– / {3,4,5,6,7,8}	0.0060	– / {3,4,5,6,7,8}	0.0112	– / {3,4,5,6,7,8}
(3) VPT1	0.0065	{1,2,4,8} / {5,6,7}	0.0025	{1,2,4} / {5,6,7,8}	0.0068	{1,2,4,8} / {5,6,7}
(4) VPT2	0.0074	{1,2} / {3,5,6,7}	0.0033	{1,2} / {3,5,6,7,8}	0.0077	{1,2} / {3,5,6,7}
(5) RFP	0.0055	{1,2,3,4,8} / –	**0.0016**	{1,2,3,4,7,8} / –	0.0056	{1,2,3,4,8} / –
(6) RFD	0.0055	{1,2,3,4,8} / –	0.0016	{1,2,3,4,8} / –	0.0056	{1,2,3,4,8} / –
(7) RM	**0.0054**	{1,2,3,4,8} / –	0.0017	{1,2,3,4} / {5}	**0.0055**	{1,2,3,4,8} / –
(8) RSC	0.0078	{1,2} / {3,5,6,7}	0.0023	{1,2,3,4} / {5,6}	0.0080	{1,2} / {3,5,6,7}

Results for stochastic part estimation (MRS) for EEX

Table 3: Mean Squared Error (MSE) for estimation of **base regime parameters** for the MRS model for the conducted simulation study of the EEX market.

	EEX (Wavelet, Base Regime)					
Measure	MSE (α_1)	Worse / Better	MSE (α_1/β_1)	Worse / Better	MSE (σ_1)	Worse / Better
(1) ORG	0.0616	– / All	0.2530	– / $\{2,3,4,5,7,8\}$	4.5006	– / All
(2) FPT	0.0357	$\{1,6\}$ / $\{4,8\}$	0.1441	$\{1,6\}$ / $\{4\}$	2.7430	$\{1\}$ / $\{3\}$
(3) VPT1	0.0291	$\{1,6,7\}$ / –	0.1087	$\{1,6,7\}$ / –	**2.1131**	$\{1,2\}$ / –
(4) VPT2	0.0236	$\{1,2,5,6,7\}$ / –	0.0917	$\{1,2,5,6,7,8\}$ / –	3.2302	$\{1\}$ / –
(5) RFP	0.0311	$\{1,6,7\}$ / $\{4\}$	0.1268	$\{1,6,7\}$ / $\{4\}$	2.5537	$\{1\}$ / –
(6) RFD	0.0450	$\{1\}$ / $\{2,3,4,5,8\}$	0.1847	– / $\{2,3,4,5,7,8\}$	2.4833	$\{1\}$ / –
(7) RM	0.0368	$\{1\}$ / $\{3,4,5,8\}$	0.1498	$\{1,6\}$ / $\{3,4,5,8\}$	3.1372	$\{1\}$ / –
(8) RSC	**0.0185**	$\{1,2,6,7\}$ / –	**0.0822**	$\{1,6,7\}$ / $\{4\}$	2.3781	$\{1\}$ / –

Summary of results

Table 4: Summary of overall performance for the considered outlier detection methods based on MSE results for both markets and LTSC estimation techniques. The best results are emphasized in bold, the worst are underlined.

	Seasonal pattern	Stochastic component
Measure	Average rank	
(1) ORG	7.08	3.27
(2) FPT	4.83	2.51
(3) VPT1	3.24	1.70
(4) VPT2	6.44	1.75
(5) RFP	**1.09**	1.98
(6) RFD	1.33	2.39
(7) RM	1.36	2.04
(8) RSC	2.51	**1.63**

Each method is assigned a number (a 'rank') **= 1 + the number of methods performing significantly better**

Conclusions

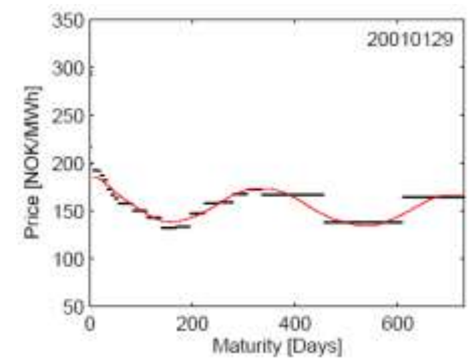
- Estimates of the seasonal pattern based on filtered data are significantly better than based on ORG
  - Best results for seasonal pattern estimation obtained for recursive filter on prices (RFP) or price differences (RFD) or a recursive seasonal model (RM) estimation
- Results are not that clear-cut for estimation of parameters of stochastic process
  - … but still using no filter generally yields the worst results
  - Here, MRS model classification (RSC) and variable price thresholds (VPT1, VPT2) tend to provide best results

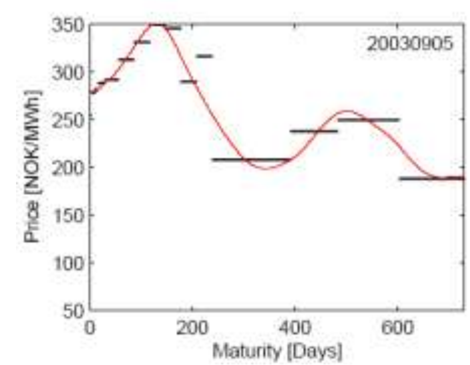
Agenda

- Introduction
- Case study I: Spikes and seasonality
- **Case study II: Forecasting LTSC**
  - J.Nowotarski, J.Tomczyk, R.Weron (2012) *Robust estimation and forecasting of the long-term seasonal component of electricity spot prices*, Work in progress

Forecasting the LTSC

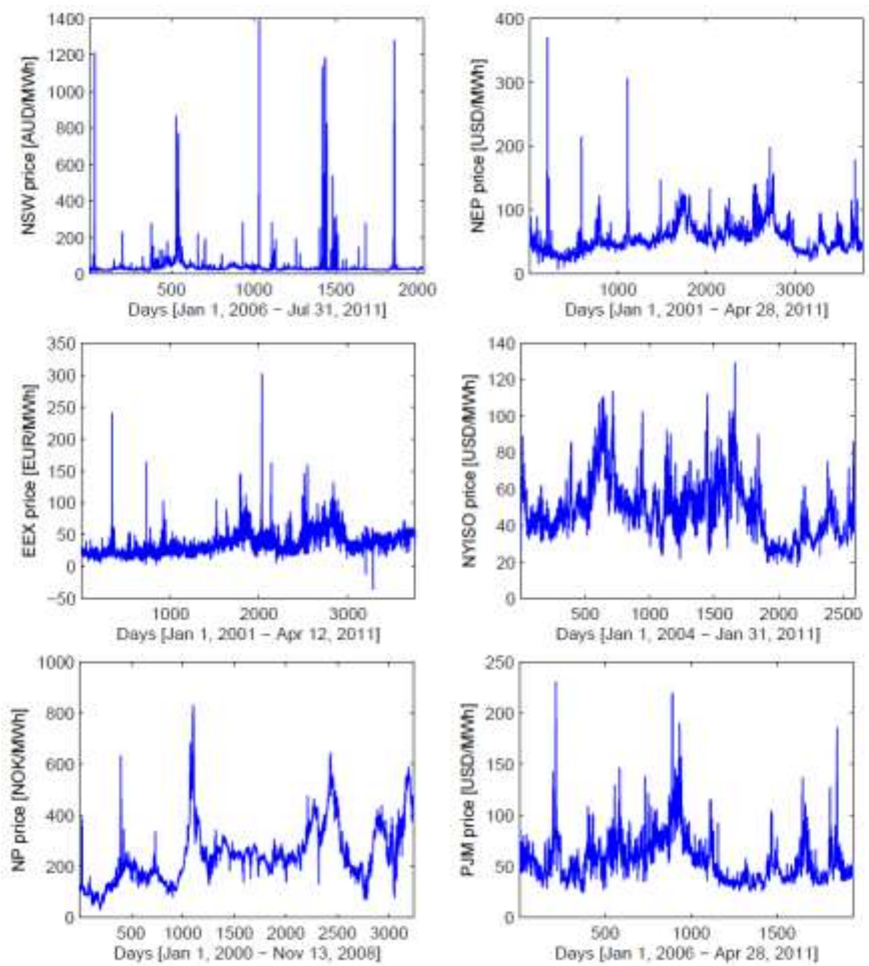
- Piecewise constant functions or sinusoids
  - Is the seasonal component really periodic?
- Wavelets
  - How to extrapolate the smoothed price?
  - What about the detail coefficients?
- Forward prices
  - Smooth interpolation of forward prices
  - What about the far end?
  - What about risk premia?





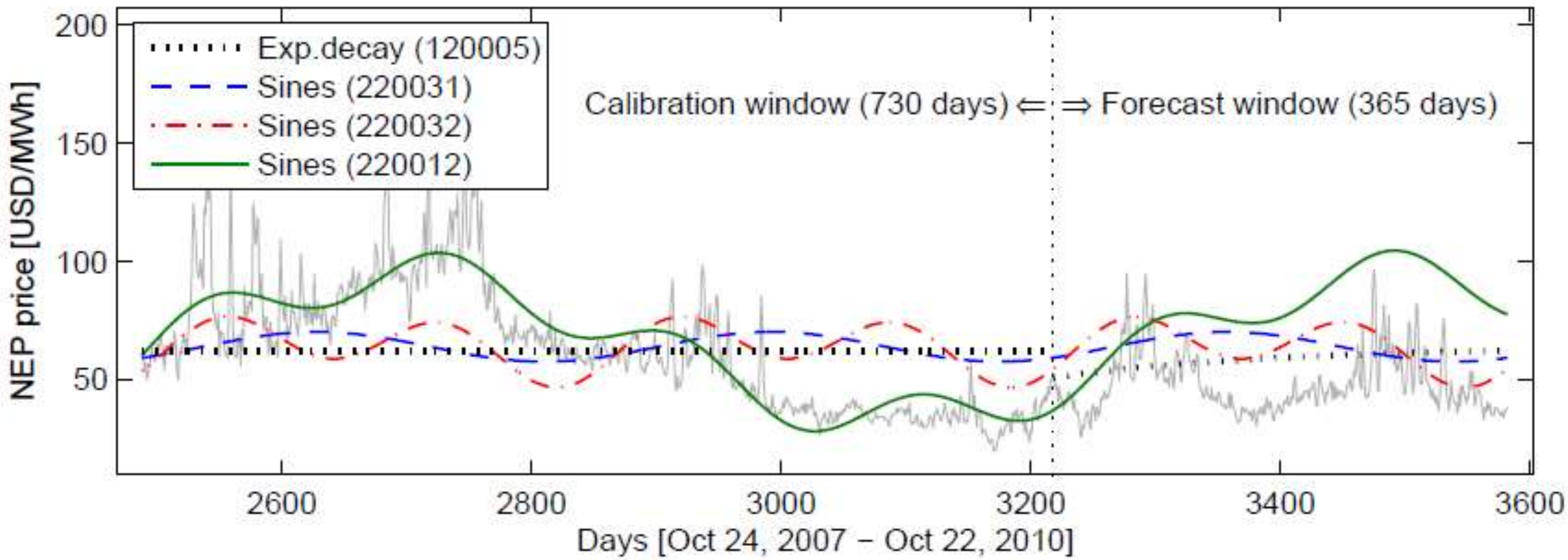
Case study

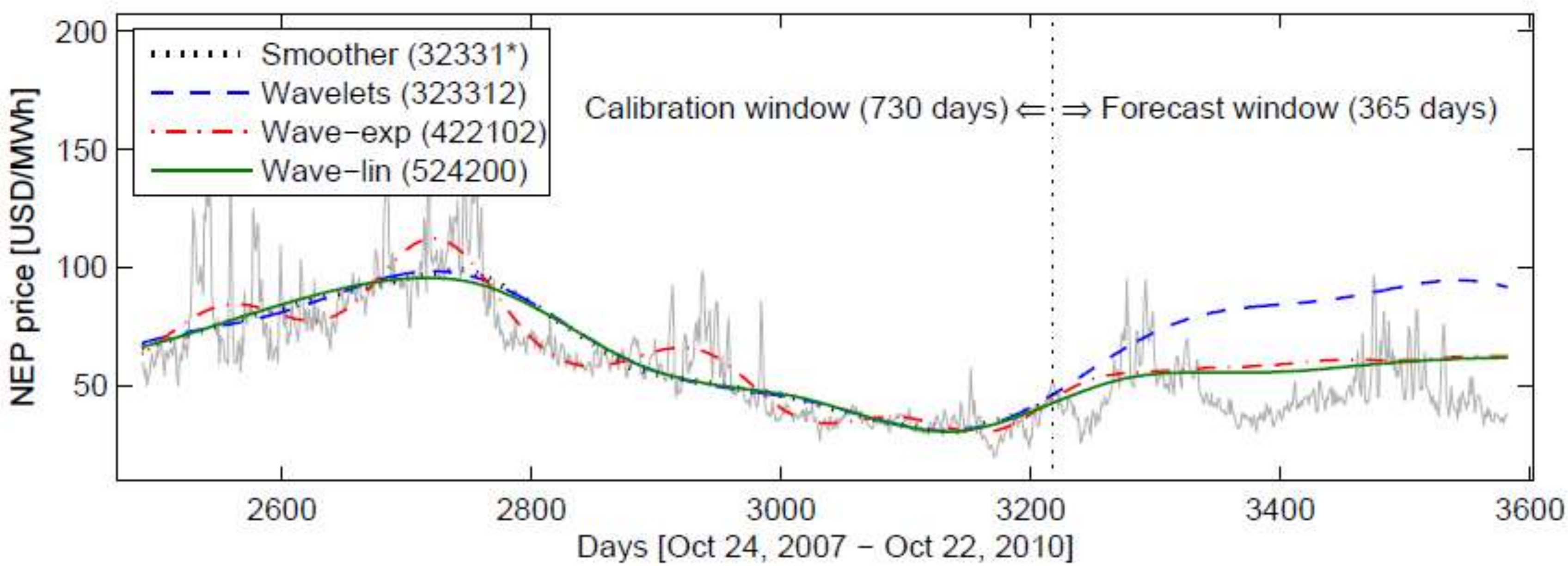
- 6 markets, daily data:
    - 578 daily forecasts for **NSW**, 2294 (**EEX**), 1780 (**Nord Pool**), 2310 (**NEPOOL**), 1128 (**NYISO**), 484 (**PJM**)
- 2 rolling calibration windows:
    - 2 year (730 day)
    - 3 year (1095 day)
- 365 day forecasting window with 6 `sub-windows':
    - 1-7d, 8-30d, 31-90d
    - 91-182d, 183-274d, 275-365d

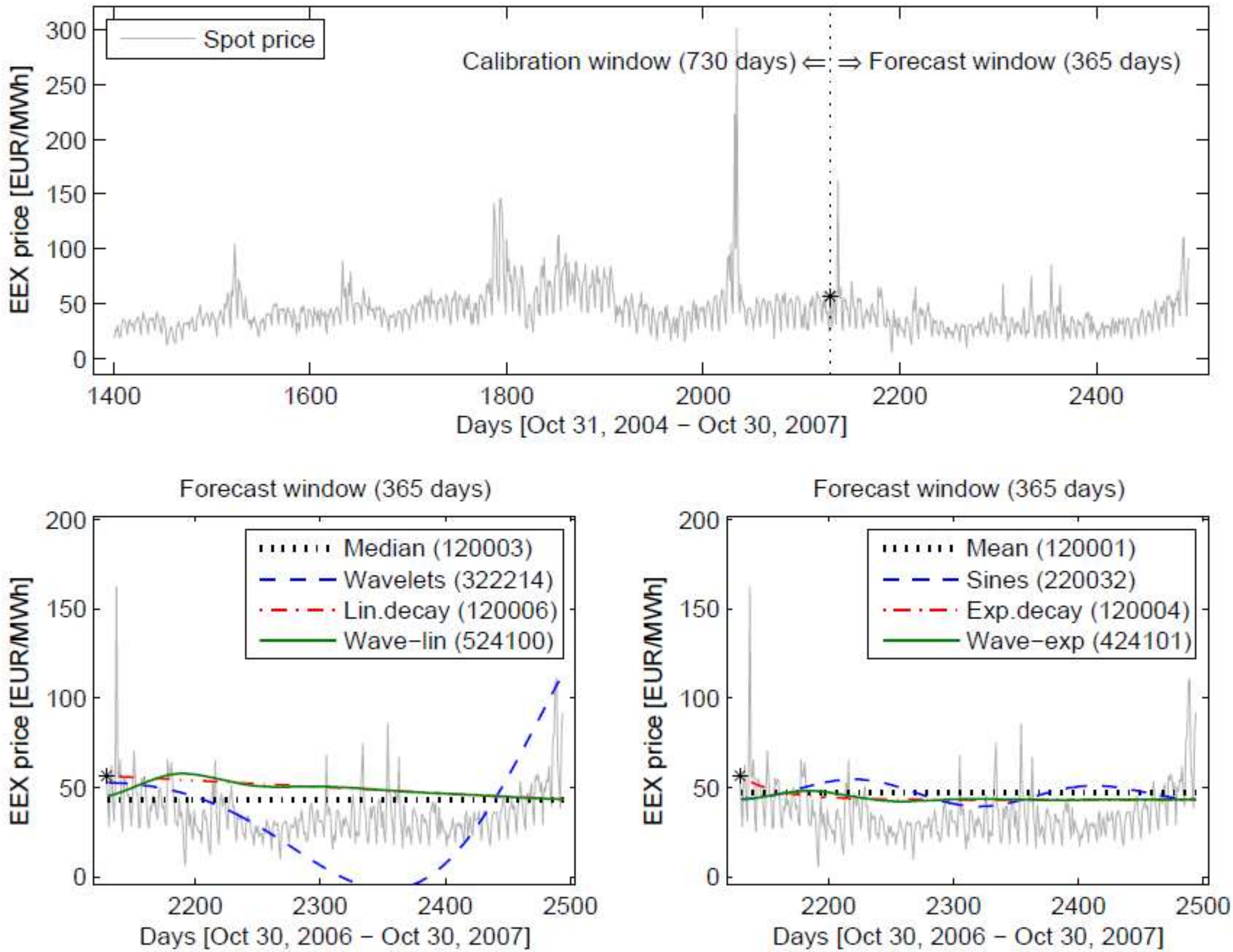


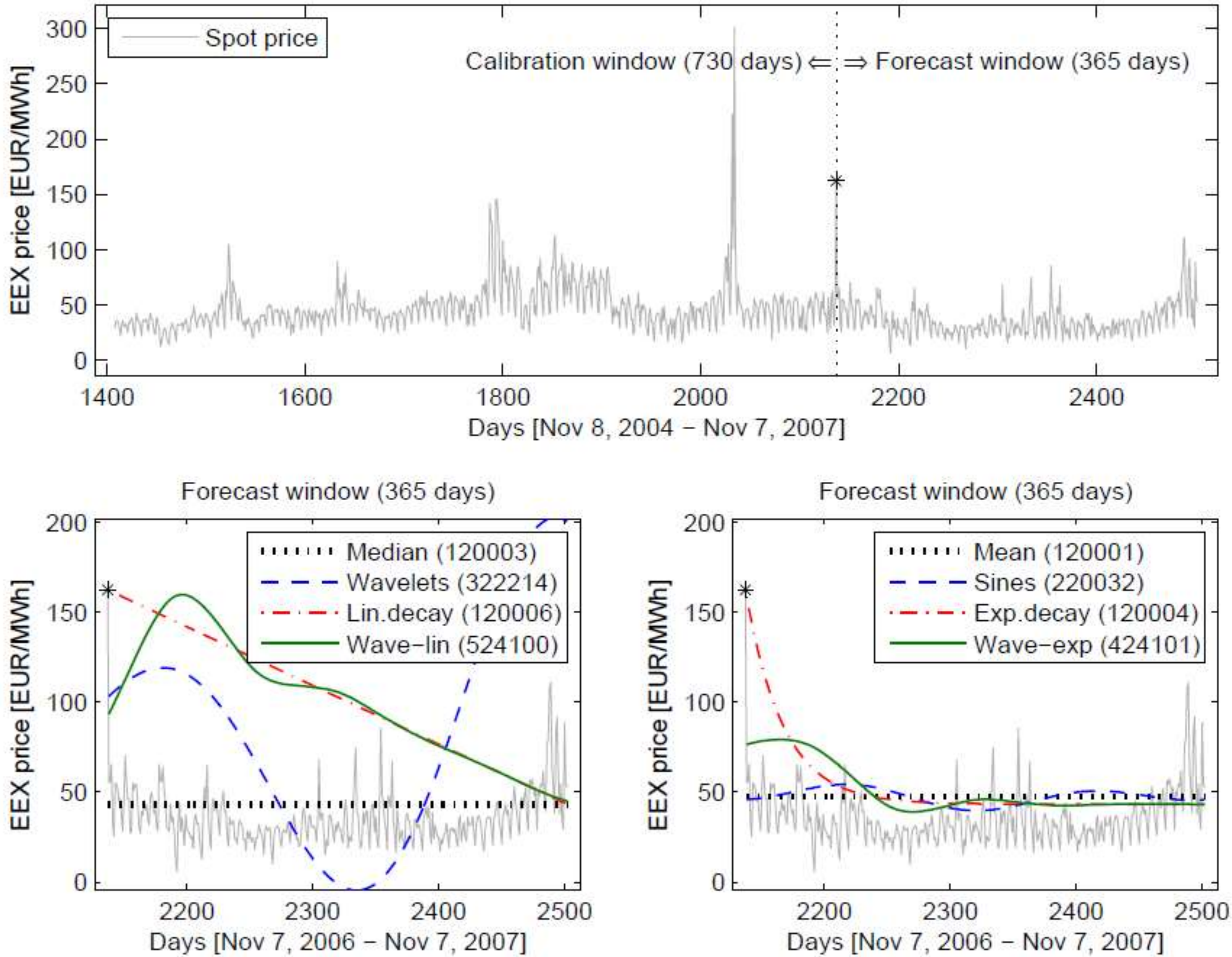
Case study: 300 models

- Simple models (1*000*)
  - Mean, linear regression, median, exponential (2) & linear decay to the median
- Sines (2*00**)
  - Up to 4 sines with fixed (1, 0.5, 0.33, 0.25 Y) or estimated periods
- Wavelets (3*****)
  - 4 wavelets: db12, db24, coif2, coif4
  - 3 wavelet approximation levels: 6, 7, 8
  - 2 extension schemes (to the next power of 2): sp0, sp1
  - Extrapolation: up to 4 sines fitted to wavelet smoothers S_6 (S_7, S_8)
- Wavelets fitted to an exponentially (4***0*) or a linearly (5***00) decaying function to the median


Exp.decay (120005)
Sines (220031)
Sines (220032)
Sines (220012)
Calibration window (730 days) ⇐ ⇒ Forecast window (365 days)
NEP price [USD/MWh]
200
150
100
50
2600
2800
3000
3200
3400
3600
Days [Oct 24, 2007 − Oct 22, 2010]


Smoother (32331*)
Wavelets (323312)
Wave−exp (422102)
Wave−lin (524200)
Calibration window (730 days) ⇐ ⇒ Forecast window (365 days)
NEP price [USD/MWh]
200
150
100
50
2600
2800
3000
3200
3400
3600
Days [Oct 24, 2007 − Oct 22, 2010]


Spot price
Calibration window (730 days) ⇐ ⇒ Forecast window (365 days)
EEX price [EUR/MWh]
Days [Oct 31, 2004 − Oct 30, 2007]
Forecast window (365 days)
Median (120003)
Wavelets (322214)
Lin.decay (120006)
Wave−lin (524100)
Days [Oct 30, 2006 − Oct 30, 2007]
Forecast window (365 days)
Mean (120001)
Sines (220032)
Exp.decay (120004)
Wave−exp (424101)
Days [Oct 30, 2006 − Oct 30, 2007]


Spot price
Calibration window (730 days) ⇐ ⇒ Forecast window (365 days)
EEX price [EUR/MWh]
Days [Nov 8, 2004 – Nov 7, 2007]
Forecast window (365 days)
Median (120003)
Wavelets (322214)
Lin.decay (120006)
Wave–lin (524100)
EEX price [EUR/MWh]
Days [Nov 7, 2006 – Nov 7, 2007]
Forecast window (365 days)
Mean (120001)
Sines (220032)
Exp.decay (120004)
Wave–exp (424101)
EEX price [EUR/MWh]
Days [Nov 7, 2006 – Nov 7, 2007]

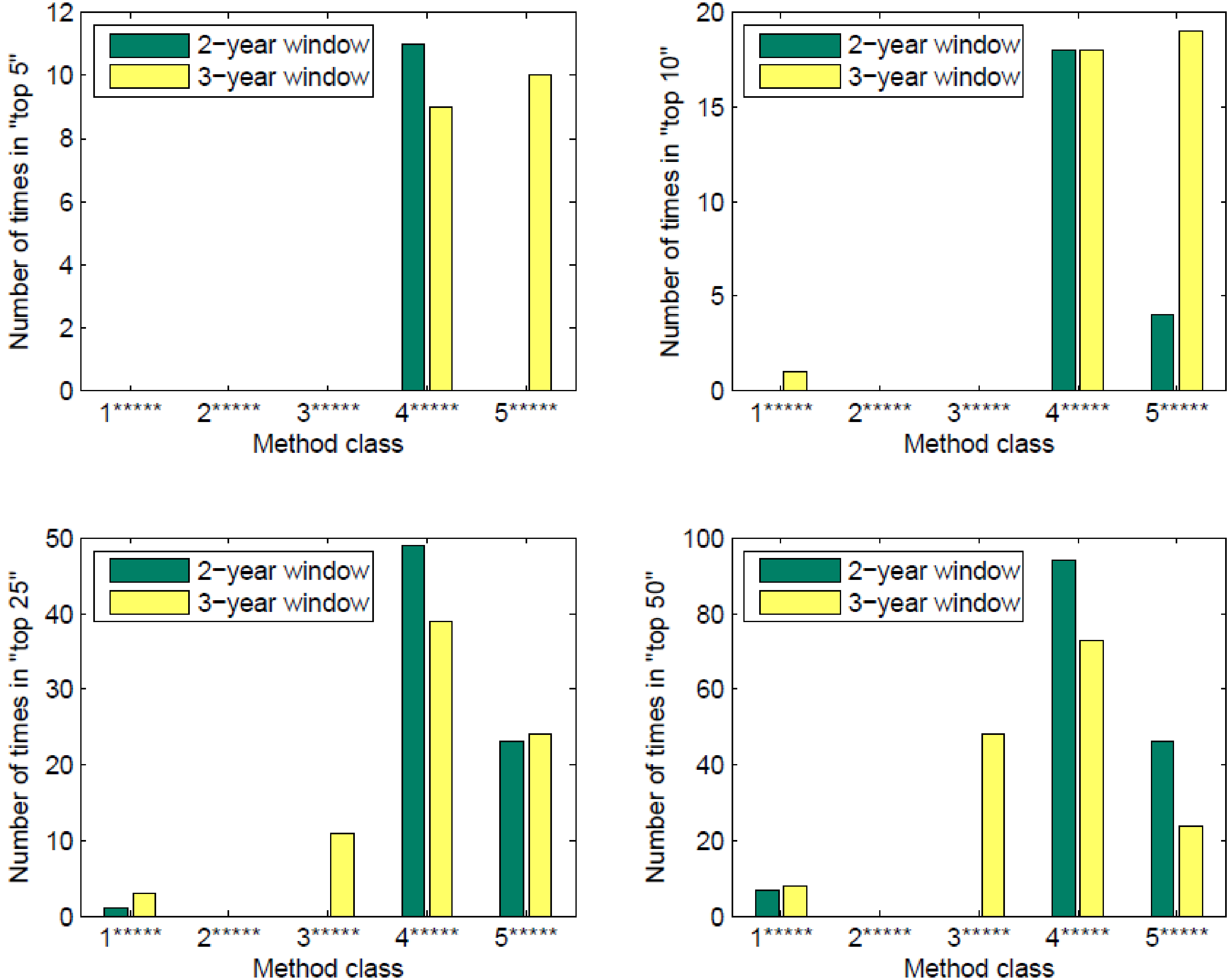


Figure 6: Histograms based on $GM(MSE_{h,*})$

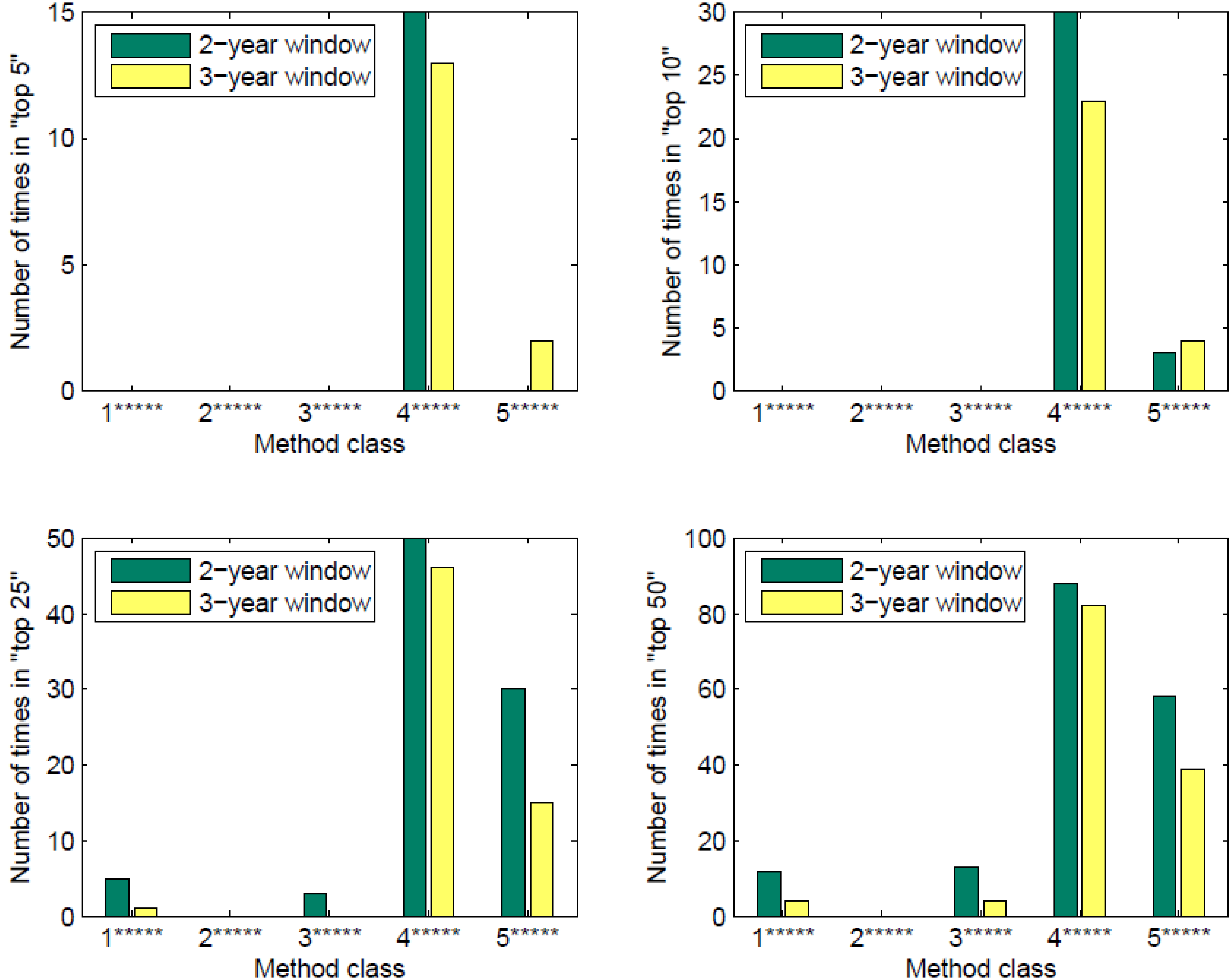


Figure 7: Histograms based on $MAPE_{h,*}$

Table 3: Top 20 models according to the three global forecast error measures: GM(MAE$_{*,*}$) in columns 2-3, GM(MAE$_{*,*}$) in columns 4-5 and MAPE$_{*,*}$ in columns 6-7. See Section 4.2 for model codes and Section 5 for error measure definitions.

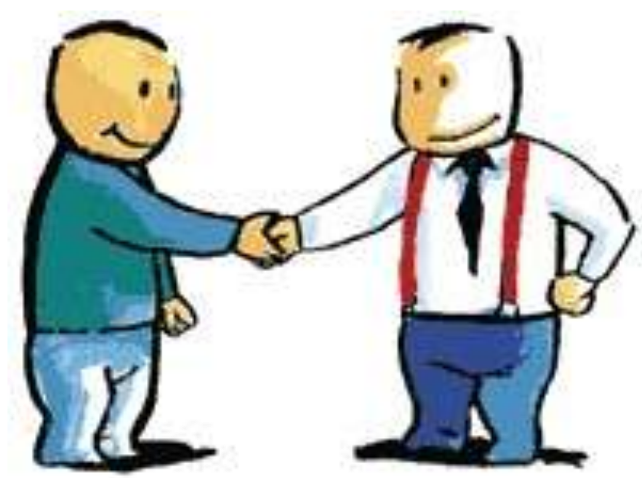
No.	GM(MAE$_{*,*}$)	Model	GM(MSE$_{*,*}$)	Model	MAPE$_{*,*}$	Model
1	8.09	**423302**[1]	5.98	**424302**[2]	29.67%	**423302**[1]
2	8.88	423102	7.69	422302	29.67%	**424302**[2]
3	11.53	524300	8.45	**423302**[1]	29.80%	422302
4	11.64	**424302**[2]	8.72	431302	29.81%	431302
5	12.7[illegible]	431302	16.38	421302	29.91%	432302
6	12[illegible]	522300	17.56	423102	29.97%	421302
7	[illegible]	424102	20.07	421102	29.99%	434302
8	[illegible]6	523300	20.34	434302	30.16%	433302
9	[illegible]28	422302	20.85	424102	30.16%	433102
10	[illegible].28	120005	21.67	422202	30.17%	434102
11	[illegible]5.96	422102	22.01	432102	30.22%	524300
12	[illegible]	421102	22.38	424202	30.23%	431202
13	[illegible]	521300	22.48	522300	30.27%	522300
14	[illegible]	422202	22.57	523300	30.27%	431102
15	[illegible]	521200	22.73	421202	30.31%	432202
16	[illegible]	433102	23.63	432302	30.34%	432102
17	[illegible]	523200	23.87	422102	30.34%	523300
18	[illegible]	421302	24.05	524200	30.36%	434202
19	20.01	522200	24.13	431102	30.38%	433202
20	20.28	424202	24.25	524300	30.39%	531300

423302**:** Wavelets with
exponential decay (**4**),
2Y calibration window (**2**),
\`coif2' wavelet (**3**),
S_8 approx. (**3**),
slow (\`180 day') decay (**2**)

Conclusions

- Simple models are surprisingly good for long-term (> ½ year) forecasts
- Sinusoidal models yield poor in-sample and/or out-of-sample fits
- Wavelets with decay to the median win in nearly all statistics
- *Future*: Compare with forward price forecasts

The end



References

- J. Janczura, S. Trück, R. Weron, R. Wolff (2012) Identifying spikes and seasonal components in electricity spot price data: A guide to robust modeling, RePEc/SSRN Paper
- J. Janczura, R. Weron (2010) An empirical comparison of alternate regime-switching models for electricity spot prices, Energy Economics 32, 1059-1073
- J. Janczura, R. Weron (2012) Efficient estimation of Markov regime-switching models: An application to electricity spot prices, AStA 96(3), 385-407
- J. Nowotarski, J. Tomczyk, R. Weron (2012) Robust estimation and forecasting of the long-term seasonal component of electricity spot prices, Work in progress … soon on RePEc

- Reviews

  - F.E. Benth, J.S. Benth, S. Koekebakker (2008) *Stochastic Modeling of Electricity and Related Markets*, World Scientific
  - M. Burger, B. Graeber, G. Schindlmayr (2007) *Managing Energy Risk*, Wiley
  - A. Eydeland, K. Wolyniec (2012) *Energy and Power Risk Management*, Wiley
  - R. Weron (2006) *Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach*, Wiley