House of Energy Markets & Finance

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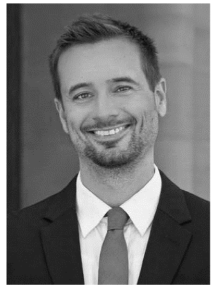
Environmental Economics and Renewable Energies	Energy Economics	Energy Trading and Finance	Macroeconomics	Finance
				
Junior Prof. Dr. Florian Ziel	Prof. Dr. Christoph Weber	Prof. Dr. Rüdiger Kiesel	Prof. Dr. Ansgar Belke	Prof. Dr. Heiko Jacobs

Our Vision: To become the leading Research Institution at the Intersection of Energy Economics and Finance

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- Analysis of market and price developments
- Investment & financing strategies for new energy technologies
- Evaluation of innovative trading products & strategies
- Interdependencies of financial and energy markets
- Sustainable, future-proof market designs
- Business models for the retail market

HEMF – Mission statement

- We research at the interface of energy economics and finance.
- We build on a unique connection of energy economics and finance methods.
- We establish a platform for the economic analysis and monitoring of the energy system transformation.
- We support young researchers in energy economics and finance as well as interdisciplinary cooperation.
- We establish a centre with international visibility and play a leading role in research on energy economics and finance.

Interface of energy economics and finance

Support young researchers

Energy system transformation

Leading role in research on energy economics and finance

Interdisciplinary cooperation

International visibility

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Modeling market order arrivals on the intraday power market for deliveries in Germany with Hawkes processes with parametric kernels

Science meets Social Science Seminar

Graf von Luckner, Kiesel | Chair for Energy Trading and Finance | University of Duisburg-Essen

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Why modeling market order arrivals?

- When studying markets from an intraday perspective, the times when market orders arrive play an important role.
- One reason for this is that market orders may change the mid price and bid-ask spread; so modeling their dynamics may go along with modeling market order arrivals.
- Algorithmic trading strategies may be designed such that the nature of market orders including their arrival times have an impact on the optimal behavior.

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Spot power market for deliveries in GER

- Contracts with hourly and quarter hourly delivery; since end of March 2017 also half hourly contracts.
- For hourly contracts, there is a day-ahead auction and continuous trading until half an hour before delivery start:

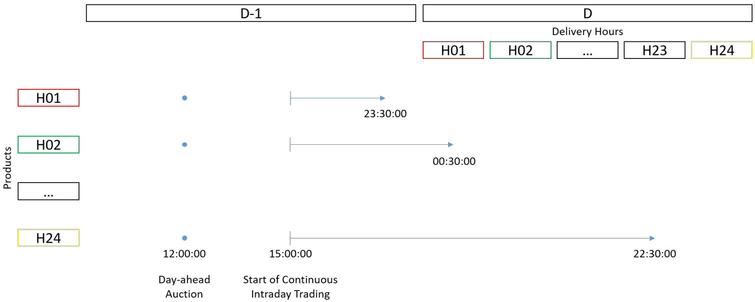

Figure: Schema of GER spot power market.

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Orders

- A buy (sell) limit order (LO) is an instrument which allows an agent to express how much she wants to pay (receive) per MWh for a specific number of MWh.
- All unfilled buy and sell limit orders are gathered in the limit order book (LOB).
- A buy (sell) market order (MO) is an instrument which allows an agent to buy (sell) a specific number of MWh at the current best sell (buy) limit order price(s).

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LOB snapshot

Order Book Details

EPEX RWE 12-13 Hi/Low: 78.64 / 65.41 Last: 10.0 @ 75.00 ACTI

VWAP	Acc	Qty	Bid
54.06	58.2	58.2	54.06
53.98	68.2	10.0	53.50
53.45	159.3	91.1	53.06
49.78	253.4	94.1	43.56
48.09	319.5	66.1	41.61
45.40	373.3	53.8	29.44
44.82	386.8	13.5	28.75
39.69	485.9	99.1	19.68
36.36	534.2	48.3	2.87

Ask	Qty	Acc	VWAP
94.10	14.0	14.0	94.10
97.36	94.0	108.0	96.94
97.86	59.2	167.2	97.26

Order Book Details*
Provides a non-aggregated view of active orders for a selected contract.

Source: `https://www.epexspot.com/document/30313/ComTrader%20-%20Guideline`

Figure: Snapshot of limit order book for product H13.

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Buy MO arrivals

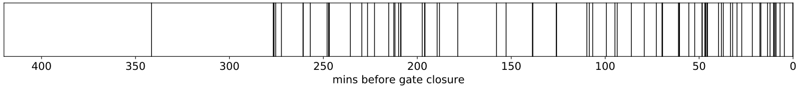

Figure: Buy MO arrivals for in the seven trading hours before gate closure for delivery start at 2015-04-07 12:00:00 UTC.

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Buy MO arrivals cont'd

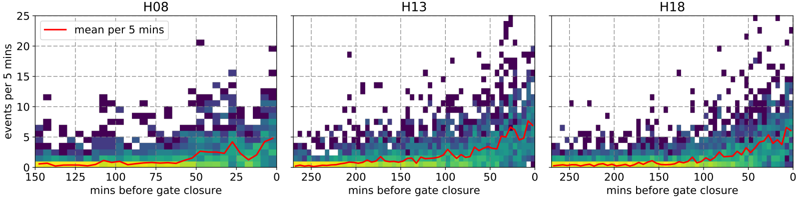

Figure: Distributions of buy MO arrivals per 5 minutes and means in Q2/2015.

Test for non-homogeneous Poisson

- Brown et al. (2005) introduce a test which may be used to investigate whether arrivals are driven by a non-homogeneous Poisson process.
- The test requires specifying a time interval L during which the arrivals may be considered to have a constant rate.
- The timestamps of all arrival events contained in time bin i, $T_{i,j}$ with $j \in \{1, \dots, J(i)\}$, are transformed according to

$$R_{i,j} = (J(i) + j - 1)\left(-\log\left(\frac{L - T_{i,j}}{L - T_{i,j-1}}\right)\right).$$

- If the arrival rate is indeed constant per specified time interval, the $\{R_{i,j}\}$ are standard exponentially distributed.

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Test results

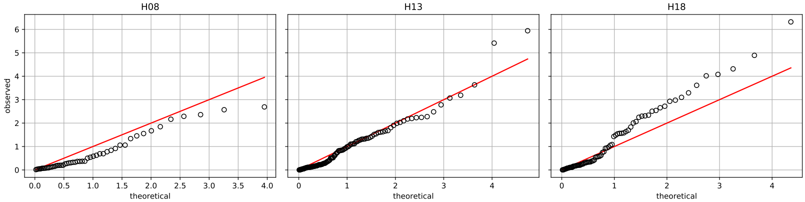

Figure: QQ plots of transformed timestamps of buy LO arrivals and the unit-rate exponential distribution for Q2/2015.

Test results cont'd

	Median(D_{91})	Median(p)	$N_{p<0.05}$		Median(D_{91})	Median(p)	$N_{p<0.05}$
H01	0.104	9.60e-02	41	H13	0.086	1.22e-03	66
H02	0.100	7.56e-02	41	H14	0.080	1.75e-03	65
H03	0.104	4.84e-02	46	H15	0.082	3.40e-03	73
H04	0.088	7.49e-02	42	H16	0.076	2.41e-03	67
H05	0.089	6.88e-02	41	H17	0.085	9.37e-04	69
H06	0.110	1.55e-02	58	H18	0.091	6.65e-04	66
H07	0.102	1.65e-02	54	H19	0.083	1.70e-03	63
H08	0.099	6.92e-03	58	H20	0.091	2.02e-03	66
H09	0.097	5.39e-03	68	H21	0.087	3.15e-03	65
H10	0.096	2.36e-03	69	H22	0.094	1.09e-03	65
H11	0.084	2.44e-03	64	H23	0.094	3.94e-04	66
H12	0.082	2.88e-03	67	H24	0.098	5.09e-04	64

Table: Results of the Kolmogorov-Smirnov test with null hypothesis that the distribution of the durations between the transformed timestamps of buy market orders in Q2/2015 and the standard exponential distribution match. $N_{p<0.05}$ is the number of times that the null hypothesis is rejected at 5% significance level.

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Share per no. of prices executed against

	Q2/2015								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	83.7%	10.8%	3.7%	1.0%	H08	84.6%	10.5%	3.4%	0.7%
H13	80.8%	11.9%	4.2%	1.6%	H13	84.1%	10.4%	3.3%	1.3%
H18	83.7%	10.8%	3.3%	1.4%	H18	82.2%	11.5%	4.1%	1.3%
	Q2/2016								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	75.7%	13.0%	6.4%	2.9%	H08	79.2%	11.4%	5.2%	2.6%
H13	80.4%	11.8%	4.6%	1.9%	H13	78.8%	12.4%	5.4%	2.0%
H18	82.2%	11.1%	4.1%	1.7%	H18	79.3%	12.7%	4.7%	2.0%

Table: Shares of MOs executed against $1, \ldots, 4$ price/s in all MOs.

Execution costs per no. of prices executed against

	Q2/2015								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	0.00	0.27	0.61	0.61	H08	0.00	0.47	0.63	0.81
H13	0.00	0.38	0.76	0.83	H13	0.00	0.18	0.23	0.33
H18	0.00	0.13	0.26	0.43	H18	0.00	0.12	0.28	0.38
	Q2/2016								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	0.00	0.13	0.25	0.46	H08	0.00	0.12	0.25	0.38
H13	0.00	0.09	0.20	0.29	H13	0.00	0.09	0.17	0.26
H18	0.00	0.08	0.18	0.31	H18	0.00	0.07	0.16	0.27

Table: Mean execution costs of MOs executed against $1, \ldots, 4$ price/s.

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Point process

Definition 1 (Point process)

Consider some stochastic process which has support over the whole time axis $(-\infty, \infty)$ and whose realizations are points on that axis. Let $\mathcal{F}_t$ denote the history of that process at time t. Furthermore, let $N(t)$ denote a process reflecting the number of points of the process at time t. The process is a point process if the requirements that for all t, as $h \to 0+$,

$$\mathrm{P}\left(N(t+h) - N(t) = 0 | \mathcal{F}_t\right) = 1 - \lambda(t) + o(h)$$
$$\mathrm{P}\left(N(t+h) - N(t) = 1 | \mathcal{F}_t\right) = \lambda(t)h + o(h)$$
$$\mathrm{P}\left(N(t+h) - N(t) > 1 | \mathcal{F}_t\right) = o(h),$$

where $\lambda(t)$ is non-negative, are fulfilled.

Hawkes process

Definition 2 (Hawkes process)

Some point process is a Hawkes process if its conditional intensity function $\lambda(t)$ has the form

$$\lambda(t) = \mu(t) + \int_0^t \phi(t-u)\, dN(u),$$

where $\mu(t)$ is called the baseline intensity function, $\phi(t)$ is called the excitement function or kernel and $N(t)$ is the counting process associated with the point process.

The previous two definitions are based on Cox and Isham (1980).

Some literature

- Jain and Joh (1988) is an example for an early work in which daily periodicities are found in trading volume data.
- In Engle and Russell (1998) the so-called autoregressive conditional duration (ACD) model for durations between consecutive events is proposed; it provides for daily periodicities and stochastic event clustering.
- Bowsher (2007) is an early work in which the Hawkes counting process which involves self-excitement is used to model financial market events.

Some literature cont'd

- In Rambaldi et al. (2017) differences in self-excitement for groups of market orders with different volumes and their interaction are studied.
- We consider that approach to be interesting in the context of algorithmic trading on the intraday power market, the reason being that thus market peculiarities such as market participants placing 0.1 MW market orders in order to control their order-to-trade ratio may be accounted for (at least to some extent).

Conditional intensity

Let $(\boldsymbol{\lambda}(t))_{0\leq t\leq T}$ denote a dim d vector of conditional intensities of point processes, each of which reflects a partition of market order arrivals. We assume

$$\boldsymbol{\lambda}(t) = \boldsymbol{\mu}(t) + \boldsymbol{\phi}(t) \cdot \mathbf{1},$$

where

$$\boldsymbol{\mu}(t) = \begin{pmatrix} \mu_1(t) \\ \vdots \\ \mu_d(t) \end{pmatrix}, \quad \boldsymbol{\phi}(t) = \begin{pmatrix} \sum_{t_i^{(1)}<t} \phi_{11}(t - t_i^{(1)}) & \cdots & \sum_{t_i^{(J)}<t} \phi_{1d}(t - t_i^{(d)}) \\ \vdots & \ddots & \vdots \\ \sum_{t_i^{(1)}<t} \phi_{d1}(t - t_i^{(1)}) & \cdots & \sum_{t_i^{(J)}<t} \phi_{dd}(t - t_i^{(d)}) \end{pmatrix},$$

and $\mathbf{1}$ is a dim d vector of ones. $(\boldsymbol{N}(t))_{0\leq t\leq T}$ is a dim d vector comprising the counting processes associated with the Hawkes processes.

Baseline intensity and excitement function

For the baseline intensities we assume the following model:

$$\mu_j(t) = \gamma_j e^{\delta_j t},$$

where $\gamma_j > 0, \delta_j \geq 0$. We assume all kernels to be of exponential form, i.e.

$$\phi_{jk}(t) = \alpha_{jk} e^{-\beta_{jk} t},$$

where $\alpha_{jk} \geq 0, \beta_{jk} > 0$. In what follows, we use $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ to denote matrices comprising all α_{jk} and β_{jk}, respectively, i.e.

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1d} \\ \vdots & \ddots & \vdots \\ \alpha_{d1} & \cdots & \alpha_{dd} \end{pmatrix}, \qquad \boldsymbol{\beta} = \begin{pmatrix} \beta_{11} & \cdots & \beta_{1d} \\ \vdots & \ddots & \vdots \\ \beta_{d1} & \cdots & \beta_{dd} \end{pmatrix}.$$

Goodness-of-fit

Let $\boldsymbol{\Lambda} = (\boldsymbol{\Lambda}(t))_{0 \leq t \leq T}$ denote the compensator of $\boldsymbol{N}$, i.e.

$$\boldsymbol{\Lambda}(t) = \int_0^t \boldsymbol{\lambda}(u) du.$$

By $\tilde{\boldsymbol{N}} = (\tilde{\boldsymbol{N}}(t))_{0 \leq t \leq T}$ we denote a counting process which is a transformation of $\boldsymbol{N}$, specifically

$$\tilde{\boldsymbol{N}}(t) = \boldsymbol{N}(\boldsymbol{\Lambda}^{-1}(t)).$$

The random time change theorem says that $\tilde{\boldsymbol{N}}$ is a unit-rate Poisson process. It may be applied in practice by testing whether durations between consecutive transformed timestamps are i.i.d. unit-rate exponentially distributed.

Model selection

- One may ask whether each group of market orders should have an impact on themselves and the other groups or whether other constellations are more suited.
- Assuming $d = 2$, an example would be that the first group is impacted by itself and the second group whereas the second group is only impacted by itself.
- If some group k has no impact on group j, then $\alpha_{jk} = 0$, $\beta_{jk} \to \infty$.

A natural question is which model is the best.

Approach

- A popular model selection method is to compare Akaike Information Criterion which is asymptotically equivalent to leave-one-out cross validation (LOOCV).
- Cross validation requires the data to be i.i.d. and hence stationary, see e.g. Arlot and Celisse (2010).
- We only consider model selection on the basis of out-of-sample point forecasts under quadratic loss function.

Handling non-stationarity

Proposition 1

Let $\mathcal{F}_t$ denote the available information at time t. Furthermore, let $t_{N(t)+1}$ denote the time of the next jump of some counting process $N(t)$ with conditional intensity $\lambda(t)$ and $\tau_{N(t)+1}$ the compensation of $t_{N(t)+1}$. The optimal forecast of $\tau_{N(t)+1}$ under the quadratic loss function then is

$$\hat{\tau}^*_{N(t)+1} = \Lambda(\hat{t}^*_{N(t)+1}),$$

where $\Lambda(t)$ is the compensator of $\lambda(t)$ and $\hat{t}^*_{N(t)+1}$ is the optimal forecast of $t_{N(t)+1}$ under the quadratic loss function.

Scheme

- Consider some point in time $T_e = T_f$ and estimate all potential models for the partitions with data up to T_e.
- For each combination of these models which appears promising on the basis of goodness-of-fit testing perform point forecasts of the time of each partition's next event after T_f, yielding $\hat{\boldsymbol{t}}^*_{\boldsymbol{N}(T_f)+\boldsymbol{1}}$.
- Use compensator to transform $\hat{\boldsymbol{t}}^*_{\boldsymbol{N}(T_f)+\boldsymbol{1}}$, yielding $\hat{\boldsymbol{\tau}}^*_{\boldsymbol{N}(T_f)+\boldsymbol{1}}$.
- Compute forecast error.
- Increment T_f by Δ_f. As long as $T_f < T_e + \Delta_e$, repeat forecast loop.
- Once $T_f \geq T_e + \Delta_e$, increment T_e by Δ_e, estimate the models with data up to T_e and do forecast loop.
- Repeat as long as $T_f + \Delta_f$ is not greater than T.

Expected intensity

Assumption 1

β has row-wise identical components $\beta_1, \ldots, \beta_d$.

Lemma 1

Let $\boldsymbol{\lambda}(t)$ denote the intensity of a dim d Hawkes process. Consider Assumption 1 to hold and let $\bar{\beta} = \operatorname{diag}(\beta_1, \ldots, \beta_d)$. Then $\mathrm{E}\left[\boldsymbol{\lambda}(s) \middle| \mathcal{F}_t\right]$ satisfies

$$\frac{d}{ds}\mathrm{E}\left[\boldsymbol{\lambda}(s) \middle| \mathcal{F}_t\right] = (\boldsymbol{\alpha} - \bar{\beta}) \cdot \mathrm{E}\left[\boldsymbol{\lambda}(s) \middle| \mathcal{F}_t\right] + \bar{\beta} \cdot \boldsymbol{\mu}(s),$$

with $\mathrm{E}\left[\boldsymbol{\lambda}(t) \middle| \mathcal{F}_t\right] = \boldsymbol{\lambda}(t)$.

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Expected count

Lemma 2

Let $\boldsymbol{N}(t)$ denote the counting process associated with some dim d point process. Then $\mathrm{E}\left[\boldsymbol{N}(s) | \mathcal{F}_t\right]$ satisfies

$$\frac{d}{ds}\mathrm{E}\left[\boldsymbol{N}(s) | \mathcal{F}_t\right] = \mathrm{E}\left[\boldsymbol{\lambda}(s) | \mathcal{F}_t\right],$$

with $\mathrm{E}\left[\boldsymbol{N}(t) | \mathcal{F}_t\right] = \boldsymbol{N}(t)$.

Expected time of next event

Proposition 2

Let $\boldsymbol{t}_{\boldsymbol{N}(t)+\boldsymbol{1}}$ denote the times of the next arrivals of some multivariate point process with associated counting process $\boldsymbol{N}(t)$. We have that

$$\mathrm{E}\left[\boldsymbol{N}\left(\mathrm{E}\left[\boldsymbol{t}_{\boldsymbol{N}(t)+\boldsymbol{1}} \middle| \mathcal{F}_t\right]\right) \middle| \mathcal{F}_t\right] - \boldsymbol{N}(t) = \boldsymbol{1}.$$

Due to the fact that the intraday market for some delivery contract closes at some point in time, there is no infinite support for the processes representing MO arrivals. Hence, if $\mathrm{E}\left[\boldsymbol{t}_{\boldsymbol{N}(t)+\boldsymbol{1}} \middle| \mathcal{F}_t\right] > T$, we ignore that forecast.

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Simulation

- If Assumption 1 does not hold, the conditional expectations of the times of the next arrivals cannot be computed analytically as shown in the previous section.
- It is possible to resort to simulation though in order to approximate the first moment.
- We extend the algorithm suggested by Chen and Stindl (2018) in two dimensions: on the one hand, we allow the process to have a history. On the other hand, we allow the process to be multivariate.

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Split

We consider split between buy market orders which do not cause the best ask price to change (referred to as “noimp”) and buy market orders which do cause the best ask price to change (referred to as “imp”).

Goodness-of-fit

Delivery start	j	k's	$\bar{\beta}$	N	N_s	$N_{p_{KS}>0.05}$	$N_{p_{LB}>0.05}$
peak	noimp	{}	True	1092	925	171	751
		{noimp}	False	1092	941	892	825
		{imp}	False	1092	858	199	705
		{noimp,imp}	True	1092	946	894	832
		{noimp,imp}	False	1092	788	757	694
	imp	{}	True	1092	1048	349	898
		{noimp}	False	1092	976	449	848
		{imp}	False	1092	1033	1008	922
		{noimp,imp}	True	1092	1035	1009	922
		{noimp,imp}	False	1092	957	942	866

Table: Results of goodness-of-fit tests. N is the number of estimated models. N_s is the number of models which are estimated successfully. $N_{p_{KS}>0.05}$ is the number of successfully estimated models where the null hypothesis of the KS test is not rejected at 5% significance level. $N_{p_{LB}>0.05}$ is the number of successfully estimated models where the null hypothesis of the LB test for the first five lags is not rejected at 5% significance level.

Branching ratios

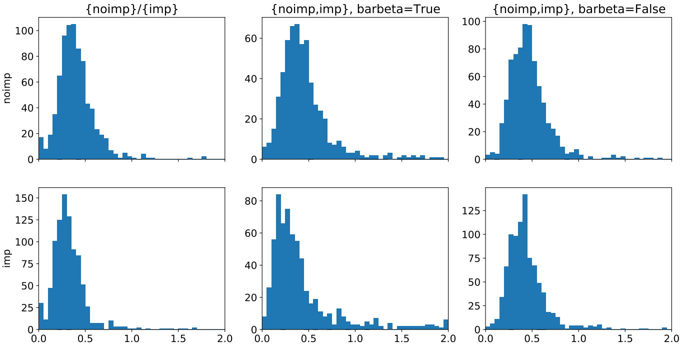

Figure: Histograms of branching ratios $\sum_k \frac{\alpha_{jk}}{\beta_{jk}}$.

Model selection

				noimp		imp	
noimp	imp	$\bar{\beta}$	N	mean	variance	mean	variance
{noimp}	{imp}	n/a	385	0.3480	0.0848	0.6453	0.7496
{noimp}	{noimp,imp}	True	385	0.3480	0.0848	0.2324	0.0658
{noimp}	{noimp,imp}	False	385	0.3480	0.0848	0.2295	0.0788
{noimp,imp}	{imp}	True	277	0.1789	0.0274	0.5434	0.4421
{noimp,imp}	{imp}	False	257	0.2122	0.0638	0.6987	0.9366
{noimp,imp}	{noimp,imp}	True	277	0.1032	0.0366	0.1919	0.0462
{noimp,imp}	{noimp,imp}	False	257	0.2137	0.0607	0.2638	0.1138

Table: For each combination of the models for the partitions which are promising according to goodness-of-fit testing mean and variance of the squared errors from forecasting the compensated time of the next event. N is the number of forecasts. Delivery start is at 2015-04-07 12:00:00 UTC.

Model selection cont'd

noimp	imp	$\bar{\beta}$	N
{noimp}	{imp}	n/a	3
{noimp}	{noimp,imp}	True	1
{noimp}	{noimp,imp}	False	2
{noimp,imp}	{imp}	True	3
{noimp,imp}	{imp}	False	3
{noimp,imp}	{noimp,imp}	True	13
{noimp,imp}	{noimp,imp}	False	4

Table: For each combination of the models for the partitions which are promising according to goodness-of-fit testing the number of times which they have the smallest sum of mean squared errors. Delivery start at 12:00:00 UTC between 2015-04-01 and 2015-04-29.

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Conclusion

- Non-homogeneous Poisson process with exponentially increasing intensity does not appear to be a promising model.
- Hawkes process with exponential baseline intensity and exponential excitement function seems to be able capture the dynamics of market order arrivals quite well.
- For the delivery contracts with delivery start at 12:00:00 UTC between 2015-04-01 and 2015-04-29, the Hawkes process with row-wise identical components in β has the smallest MSE most of the times.

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